Final Review Questions

I: Limits. Find the limits (if it exists)

a) $\lim_{x \to -1} \dfrac{2x^2 - x - 3}{x+1}$ b) $\lim_{x \to 2} \dfrac{2-x}{x^2-4}$

c) $\lim_{t \to 1} \dfrac{t^2+t-2}{t^2-1}$ d) $\lim_{x \to 2} \dfrac{|x-2|}{x-2}$

e) $\lim_{x \to 1} f(s)$ where $f(s) = \begin{cases} s, & s \le 1 \\ 1-s & s > 1 \end{cases}$

II Use the definition of derivative: $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = f'(x)$,

to find the derivative of the function:

a) $f(x) = 1 - x^2$ b) $f(x) = x^2 - 5x + 3$

III. ~~The~~ Marginal Cost: The cost (in \$) of producing x units of a product is given by: $C = 3.6\sqrt{x} + 500$

a) Find the marginal cost when $x = 9$.

b) Explain what marginal cost means.

IV. Marginal Revenue: The revenue (in \$) from renting x apartments can be modeled by

$$R = 2x(900 + 32x - x^2)$$

a) Find the marginal revenue when $x = 14$.

b) Explain what marginal revenue means.

V. Differentiate:

a) $f(x) = \sqrt[3]{x}\,(x+1)$

b) $f(x) = \dfrac{x^3+3x+2}{x^2-1}$

c) $f(x) = (x^5-3x)\left(\dfrac{1}{x^2}\right)$

d) $f(x) = \dfrac{x+1}{\sqrt{x}}$

e) $f(x) = \dfrac{4x^2-3x}{8\sqrt{x}}$

f) $g(x) = \dfrac{3}{\sqrt[3]{x^3-1}}$

g) $f(x) = \sqrt{\dfrac{3-2x}{4x}}$

h) $g(t) = \dfrac{3t^2}{\sqrt{t^2+2t-1}}$

i) $y = \left(\dfrac{6-5x}{x^2-1}\right)^2$

j) $y = \left(\dfrac{4x^2}{3-x}\right)^3$

k) $y = e^{-2x+x^2}$

l) $g(x) = e^{x^3} + x^2e^{-x} + e^{\sqrt{x}}$

m) $y = e^{-1/x^2} + \dfrac{x}{e^{2x}}$

n) $y = (e^{4x}-2)^2$

o) $y = \ln\left(x\sqrt{x^2-1}\right)$

p) $y = \dfrac{x}{x^2+1}$

q) $y = \ln\sqrt{\frac{x+1}{x-1}}$

r) $f(x) = \ln\frac{1+e^x}{1-e^x}$

s) $y = \frac{\ln x}{x^2}$

t) $g(x) = e^{-x}\ln x$

VI Find $\frac{dy}{dx}$!

a) $4x^2y - \frac{3}{y} = 0$

b) $x^2y^2 - 4y = 1$

c) $2xy^3 - x^2y = 2$

d) $\frac{xy - y^2}{y - x} = 1$

e) $xe^x + 2ye^x = 0$

f) $e^{xy} + x^2 - y^2 = 10$

g) $4xy + \ln(x^2y) = 7$

h) $4x^3 + \ln y^2 + 2y = 2x$

i) $x^2 - 3\ln y + y^2 = 10$

VII Find the slope of the graph of the given function at the given point. Write the equation of the tangent line:

a) $4x^2 + 9y^2 = 36$ at $(\sqrt{5}, \frac{4}{3})$

b) $x^2 - y^3 = 0$ at $(-1, 1)$

VIII. Find the intervals where the graph of the given function is <u>increasing / decreasing</u>.

<u>Find all relative extrema:</u>

a) $f(x) = x^4 - 32x + 4$ b) $f(x) = x^4 - 12x^3$

c) $f(s) = \dfrac{s}{s-2}$ d) $g(t) = \dfrac{t^2}{t^2+3}$

e) $f(x) = \dfrac{4x}{x^2+1}$ f) $g(x) = 6x^3 - 15x^2 + 12x$

IX Find the intervals on which the graph is <u>concave up/downward</u>. Find the <u>points of inflection</u>, (if any).

a) $y = -x^3 + 3x^2 - 2$ b) $f(x) = \dfrac{x^2-1}{2x+1}$

c) $f(x) = \dfrac{x^2+4}{4-x^2}$ d) $y = \dfrac{x^2}{x^2+1}$

e) $y = -x^3 + 6x^2 - 9x - 1$ f) $y = x^5 + 5x^4 - 40x^2$

X) A manufacturer wants to design an open box that has a square base and a surface area of 108 in^2. What dimensions will produce a box with a maximum volume?

XI. Find the length and width of the rectangle that has area 64 cm^2 and a minimum perimeter.

XII Determine the dimensions of a rectangular solid (with a square base) of a maximum volume if its surface area is 150 in^2.

XIII The demand and cost functions for a product are

$p = 36 - 4x$ and $C = 2x^2 + 6$

(a) what level of production will produce a maximum profit?

(b) what level of production will produce a minimum average cost per unit?

XIV. Find the intervals on which the demand is elastic, inelastic and of unit elasticity:

a) $p = 60 - 0.04q$, $0 \le q \le 1500$ b) $p = \sqrt{960 - q}$, $0 \le q \le 960$

XV. The demand function of a product is given by $qp^2 - q^3 = 0$. Find the elasticity of demand when $q = 10$.

XVI The demand function of a product is given by $-p^2 + qp = 0$. Find the elasticity of demand when $p = \$5$.

XVII Sketch the graph of the function. Identify domain, asymptotes; x-, y- intercepts; relative extrema, concavity.

a) $f(x) = \dfrac{x^2}{x-1}$

b) $f(x) = \dfrac{3x^2}{2(x^2+1)}$

c) $f(x) = \dfrac{2(x^2-9)}{x^2-4}$

d) $f(x) = \dfrac{-4x}{x^2+4}$

e) $f(x) = \dfrac{2x}{1+x^2}$

f) $f(x) = \dfrac{x+1}{x-1}$

XVIII Integrate:

a) $\int \left(-\frac{9}{x^4}\right) dx$

b) $\int \left(4x^3 - \frac{1}{x^2}\right) dx$

c) $\int \left(1 - \frac{1}{\sqrt[3]{x^2}}\right) dx =$

d) $\int 3x^2\sqrt{x^3+1}\, dx$

e) $\int x(1-2x^2)^3 dx$

f) $\int \frac{x+1}{(x^2+2x-3)^2} dx$

g) $\int \frac{x-2}{\sqrt{x^2-4x+3}} dx$

h) $\int \frac{4y}{\sqrt{1+y^2}} dy$

i) $\int \frac{t+2t^2}{\sqrt{t}} dt$

j) $\int 9xe^{-x^2} dx$

k) $\int (2x+1)\, e^{x^2+x} dx$

l) $\int 3(x-4)\, e^{x^2-8x} dx$

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f) $\int \frac{x}{x^2+4} dx$

g) $\int \frac{x^2}{3-x^3} dx$

h) $\int \frac{x+3}{x^2+6x+7} dx$

i) $\int \frac{x^3-4x^2+3}{x-3} dx$

XIX If consumption is $\$8$ bill when income is 0, and if the marginal propensity to consume is

$$\frac{dC}{dy} = 0.3 + \frac{0.2}{\sqrt{y}} \quad \text{(in bill. of dollars)},$$

find the national consumption function.

XX "A certain "firm's marginal" cost for a product is $\overline{MC} = 6x + 60$, its marginal revenue is $\overline{MR} = 180 - 2x$, and its total cost of production of 10 items is $\$1000$.

a) Find the optimal level of production.

b) Find the profit function

c) Find the profit or loss at the optimal level of production

XXI If the monthly demand function is $p = 7230 - 5q^2$ and the supply function before taxation is $p = 30 + 30q^2$, what tax per unit will maximize the tax revenue T.

XXII If the demand and supply functions for a product are $p = 5000 - 20q - 0.7q^2$ and $p = 500 + 10q + 0.3q^2$, respectively, find the tax per unit t that will maximize the tax revenue T.