CHAPTER 12: Introducing Probability

Basic Practice of Statistics

7th Edition

Lecture PowerPoint Slides

In Chapter 12, we cover …

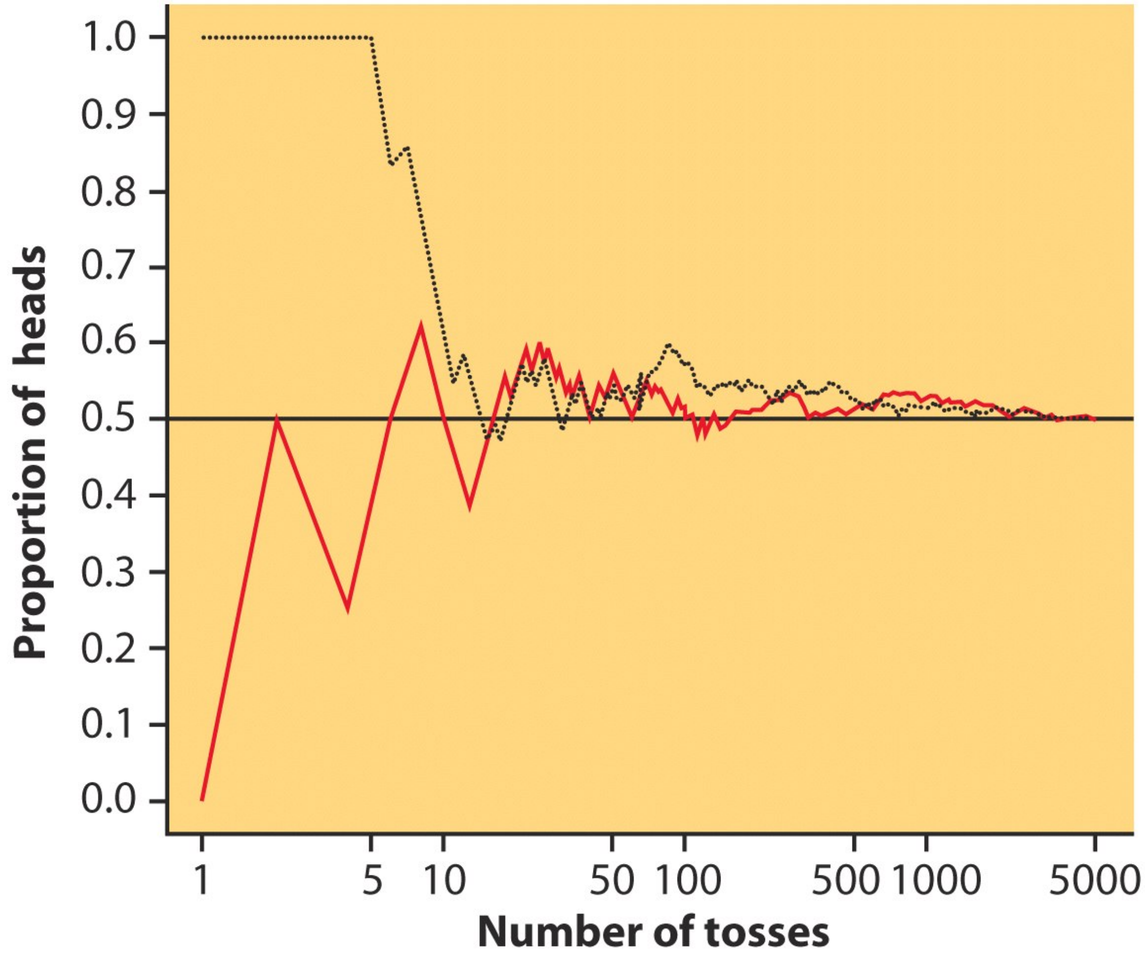
- The idea of probability
- The search for randomness
- Probability models
- Probability rules
- Finite probability models
- Continuous probability models
- Random variables
- Personal probability

Idea of Probability

- Probability is the science of chance behavior: theoretical basis for statistics
- Chance behavior is unpredictable in the short run but has a regular and predictable pattern in the long run
  - this is why we can use probability to gain useful results from random samples and randomized comparative experiments

Relative-Frequency Probabilities

Coin flipping:



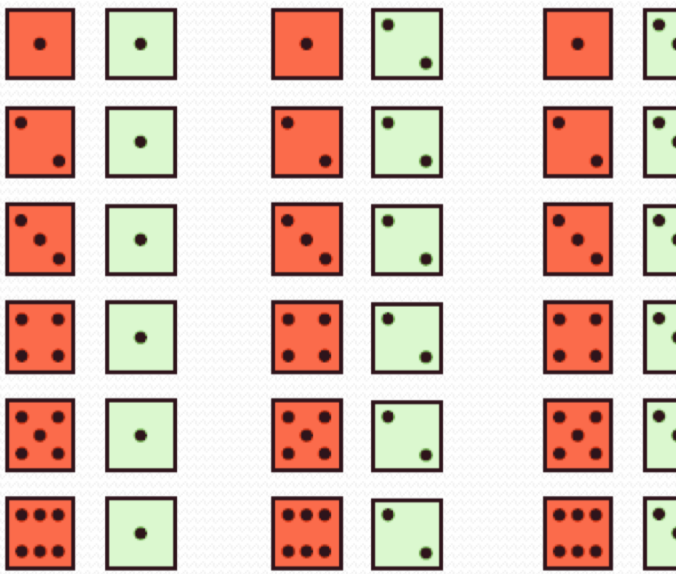
Probability models (part I)

Descriptions of chance behavior contain two parts: a list of possible outcomes and a probability for each outcome.

- The sample space S of a random phenomenon is the set of all possible outcomes.
- An event is an outcome or a set of outcomes of a random phenomenon. That is, an event is a subset of the sample space.
- A probability model is a mathematical description of a random phenomenon consisting of two parts: a sample space S and a way of assigning probabilities to events.

Probability models (part II)

Example: Give a probability model for the chance process of rolling two fair, six-sided dice—one that is red and one that is green.



Sample space
36 outcomes

Since the dice are fair, each outcome is equally likely.
Each outcome has probability 1/36.

EX1

When we toss a coin once, there are only two outcomes, heads and tails. The sample space is S={H, T}. Let A be an event that head happens for tossing a coin once. Then,

A=$\{H\}$, $P(A) = \frac{1}{2}$.

EX2

There are 36 possible outcomes when we roll two dice and record the up-faces in order (first die, second die). S={ (1,1),(1,2),$\cdots$,(6,6)}. They make up the sample space S. "Roll a 5" is an event, call it A, that contains four of these 36 outcomes: Then,

A={ (1,4),(2,3),(3,2),(4,1)}, $P(A) = \frac{4}{36} = \frac{1}{9}$

EX3

The sample space for tossing two coins is S={ (H,H), (H,T), (T,H), (T,T) }. Let A be an even that the number of heads is 1. Then, A={(H,T),(T,H)}. P(A)=2/4=1/2.

Probability Models (1 of 2)

The sample space for the experiment of flipping a coin three times is {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}. What is the probability of two tails?

a) 1/8

b) 3/8

c) 6/8

d) 7/8

Probability Models (2 of 2)

Which is the sample space for the random phenomenon of flipping a coin twice?

a) {H, T}
b) {HH, TT}
c) {HH, HT, TT}
d) {HH, HT, TH, TT}

Probability rules (part I)

1. **Any probability is a number between 0 and 1.** Any proportion is a number between 0 and 1, so any probability is also a number between 0 and 1.

2. **All possible outcomes together must have probability 1.** Because some outcome must occur on every trial, the sum of the probabilities for all possible outcomes must be exactly 1.

3. ***If two events have no outcomes in common*, the probability that one or the other occurs is the sum of their individual probabilities.**

4. **The probability that an event does not occur is 1 minus the probability that the event does occur.** The probability that an event occurs and the probability that it does not occur always add to 1, or 100%.

Probability rules (part II)

The probability rules in formal language:

Rule 1. The probability $P(A)$ of any event A satisfies $0 \leq P(A) \leq 1$.

Rule 2. If S is the sample space in a probability model, $P(S) = 1$.

Rule 3. Two events A and B are disjoint if they have no outcomes in common and, thus, can never occur together. If A and B are disjoint,

$$P(A \text{ or } B) = P(A) + P(B)$$

This is the addition rule for disjoint events.

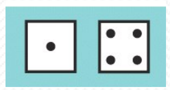
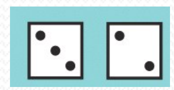
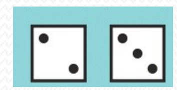
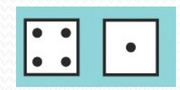
Rule 4. For any event A,

$$P(A \text{ does not occur}) = 1 - P(A)$$

Probability Rules: Mathematical Notation

Random phenomenon: roll pair of fair dice and count the number of pips on the up-faces.

Find the probability of rolling a 5.

P(roll a 5) = P() + P() + P() + P()

$= 1/36 + 1/36 + 1/36 + 1/36$

$= 4/36$

$= 0.111$

Probability rules (example)

The first digits of numbers in legitimate financial records often follow a model known as Benford's law. Call the first digit of a randomly chosen record X. Benford's law gives the following probability model for X.

First digit (X)	1	2	3	4	5	6	7	8	9
Probability	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

(a) Show that this is a legitimate probability model.

Each probability is between 0 and 1, and

$$\mathbf{0.301 + 0.176 + \cdots + 0.046 = 1}$$

(b) Find the probability that the first digit for the chosen number is not a 9.

$$\begin{aligned} \boldsymbol{P}(\textbf{not } \mathbf{9}) &= \mathbf{1 - \boldsymbol{P}(9)} \\ &= \mathbf{1 - 0.046} = \mathbf{0.954} \end{aligned}$$

Probability Rules

Let X be the number of girls in a family of three children. Does the table below show a valid assignment of probabilities to values of X?

X	0	1	2	3
Probability	0.125	0.375	0.375	0.125

a) No, because X could take on other values.

b) No, because it is not possible for X to be equal to 0.

c) Yes, because all possible values of X are included.

d) Yes, because all probabilities are between 0 and 1 and they sum to 1.

Finite probability models

- One way to assign probabilities to events is to assign a probability to every individual outcome, then add these probabilities to find the probability of any event. This idea works well when there are only a finite (fixed and limited) number of outcomes.

- A probability model with a finite sample space is called a finite probability model.

- To assign probabilities in a finite model, list the probabilities of all the individual outcomes. These probabilities must be numbers between 0 and 1 that add to exactly 1. The probability of any event is the sum of the probabilities of the outcomes making up the event.

- Finite probability models are sometimes called discrete probability models. Statisticians often refer to finite probability models as discrete.

- The Benford's law probability model was finite.

Overweight? Although the rules of probability are just basic facts about percents or proportions, we need to be able to use the language of events and their probabilities. Choose an American adult at random. Define two events:

$$A = \text{the person chosen is obese}$$

$$B = \text{the person chosen is overweight, but not obese}$$

According to the National Center for Health Statistics, $P(A) = 0.34$ and $P(B) = 0.33$.

(a) Explain why events A and B are disjoint.

(b) Say in plain language what the event "A or B" is. What is $P(A \text{ or } B)$?

(c) If C is the event that the person chosen has normal weight or less, what is $P(C)$?

Languages in Canada. Canada has two official languages, English and French. Choose a Canadian at random and ask, "What is your mother tongue?" Here is the distribution of responses, combining many separate languages from the province of Quebec:[4]

Language	English	French	Italian	Other
Probability	0.08	0.80	0.02	?

(a) What probability should replace "?" in the distribution?

(b) What is the probability that a Canadian's mother tongue is not English?

(c) What is the probability that a Canadian's mother tongue is a language other than English or French?

Finite Probability Models (1 of 6)

For the probability distribution below, what is the missing probability?

X	0	1	2	3
Probability	0.5	0.2	0.2	**?**

a) 0.9

b) 0.1

c) 1

d) 0.5

Finite Probability Models (2 of 6)

Let X be the number of girls in a family of three children. What is the probability that the number of girls is not three?

X	0	1	2	3
Probability	0.125	0.375	0.375	0.125

a) 0.125

b) 0.125

c) 0.125 + 0.375 = 0.5

d) 1 – 0.125 = 0.875

Finite Probability Models (3 of 6)

Let X be the number of girls in a family of three children. What is the probability that the family has either one or two girls?

X	0	1	2	3
Probability	0.125	0.375	0.375	0.125

a) 0.375

b) 0.375 + 0.375 = 0.75

c) 1 – 0.125 = 0.875

d) 0.5

Finite Probability Models (4 of 6)

Let *X* be the number of girls in a family of three children. What is the probability that the family has more than one girl?

X	0	1	2	3
Probability	0.125	0.375	0.375	0.125

a) 1 – 0.125 = 0.875

b) 0.375

c) 0.375 + 0.125 = 0.5

d) 0.125

Finite Probability Models (5 of 6)

A candy company manufactures bags of colorful candies. The company reports that it makes 10% each of green and red candies, and 20% each of yellow, blue, and orange candies. The rest of the candies are brown. If you pick a candy at random, what is the probability that it is brown?

a) 0.1
b) 0.2
c) 0.3
d) 0.4

Finite Probability Models (6 of 6)

A candy company manufactures bags of colorful candies. The company reports that it makes 10% each of green and red candies, and 20% each of yellow, blue, and orange candies. The rest of the candies are brown. If you pick a candy at random, what is the probability that it is either blue or orange?

a) 0.1
b) 0.2
c) 0.3
d) 0.4

Continuous probability models

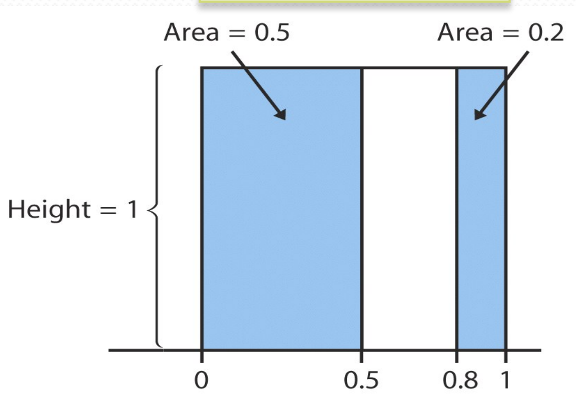
- Suppose we want to choose a number at random between 0 and 1, allowing any number between 0 and 1 as the outcome.
- We cannot assign probabilities to each individual value because there is an infinite continuum of possible values.

- A continuous probability model assigns probability as an area under a density curve. The area under the curve and above any range of values is the probability of an outcome in that range.

Example: Find the probability of getting a random number that is less than or equal to 0.5 or greater than 0.8.

$$P(X \leq 0.5 \text{ or } X > 0.8)$$
$$= P(X \leq 0.5) + P(X > 0.8)$$
$$= 0.5 + 0.2 = 0.7$$

Uniform Distribution



Normal probability models (part I)

- We can use any density curve to assign probabilities. The density curves that are most familiar to us are the Normal curves.
- Normal distributions are continuous probability models as well as descriptions of the data.
- Like all continuous probability models, the Normal assigns probability 0 to every individual outcome.

Normal probability models (part II)

- If we look at the heights of all young women, we find that they closely follow the Normal distribution, with mean $\mu = 64.3$ inches and standard deviation $\sigma = 2.7$ inches.

What is the probability that a randomly chosen young women has a height (X) between 68 and 70 inches?

$$P(68 \leq X \leq 70) = \mathrm{P}\left(\frac{68-64.3}{2.7} \leq \frac{X-64.3}{2.7} \leq \frac{70-64.3}{2.7}\right)$$
$$= \mathrm{P}(1.37 \ \leq z \ \leq 2.11)$$
$$= \mathrm{P}(z \ \leq 2.121) \ - P(z \leq 1.37)$$
$$= 0.9826 - 0.914 = 0.0679$$

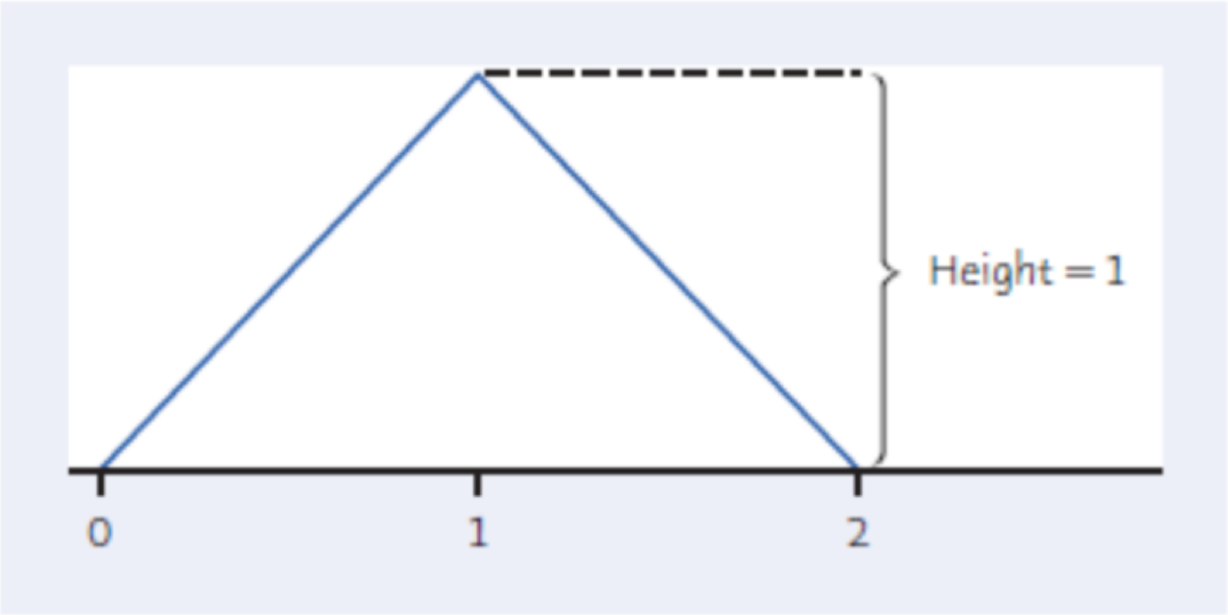
- If we repeat the random choice many times, the distribution of values of X is the same Normal distribution that describes the heights of all young women.

Adding random numbers. Generate two random numbers between 0 and 1 and take X to be their sum. The sum X can take any value between 0 and 2. The density curve of X is the triangle shown in Figure 10.7.

(a) Verify by geometry that the area under this curve is 1.

(b) What is the probability that X is less than 1? (Sketch the density curve, shade the area that represents the probability, then find that area. Do this for (c) also.)

(c) What is the probability that X is less than 0.5?



Continuous Probability Models (1 of 4)

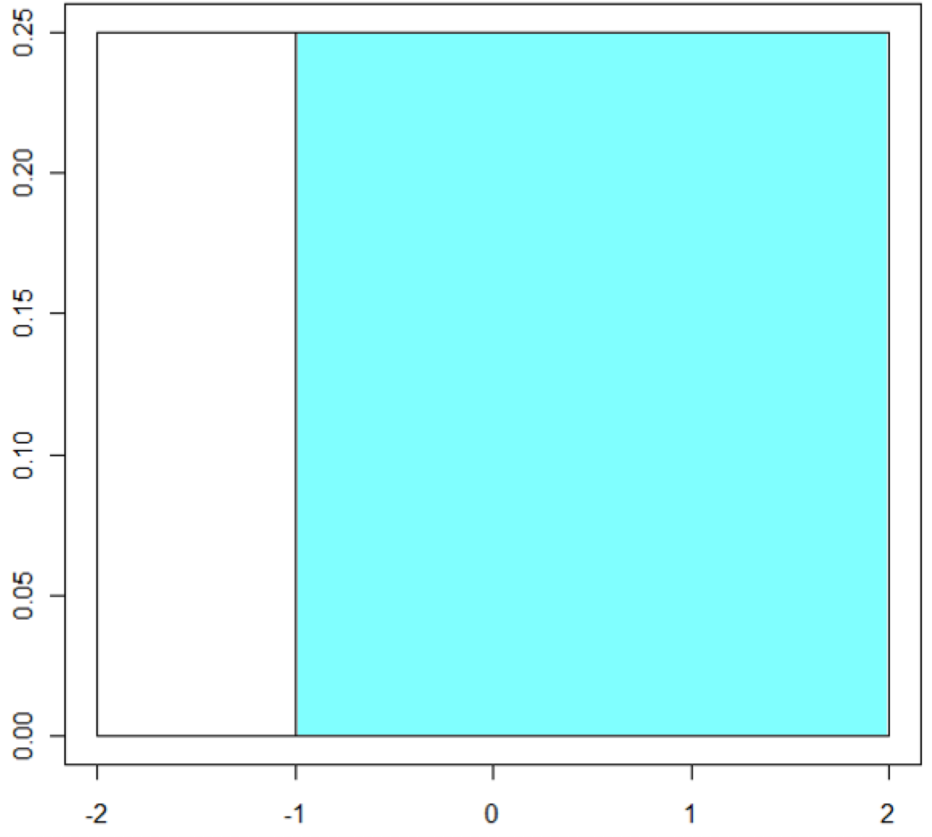
A random phenomenon results in a number in the range of –2 to +2. All numbers in the range are equally likely, so the density curve is flat. What is the height of the curve?

a) 1
b) 4
c) 0.25
d) 0.5

Continuous Probability Models (2 of 4)

A random phenomenon results in a number in the range of –2 to +2. All numbers in the range are equally likely, so the density curve is flat, as shown below. What is

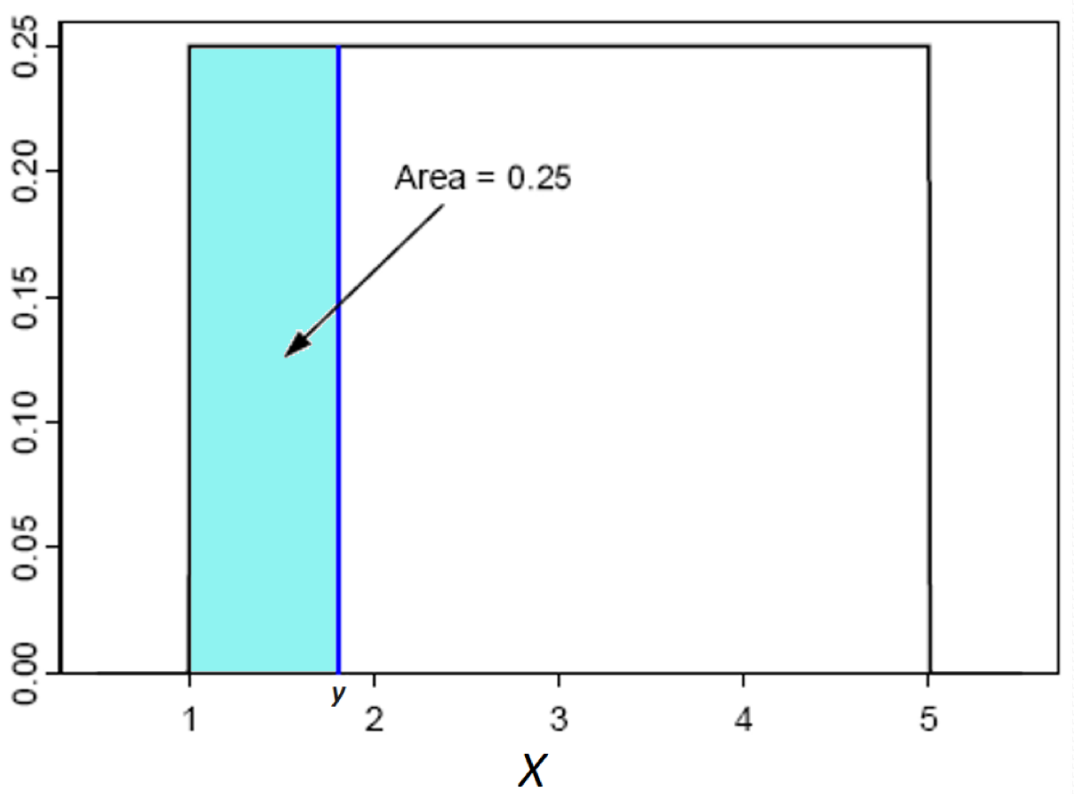
will be between –1 and +2?



a) 1
b) 0.5
c) 0.25
d) 0.75

Continuous Probability Models (3 of 4)

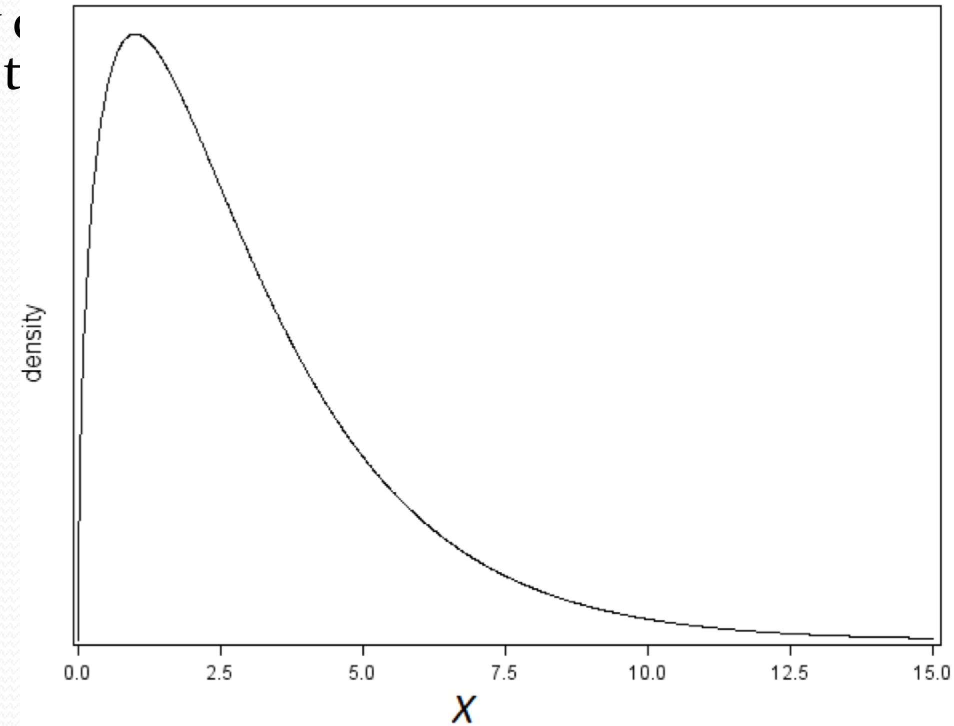
A random phenomenon results in a number in the range of 1 to 5. All values in the range are equally likely, so the density curve is flat, as shown below. What mu
number between 1 and *y* i



a) 1.5
b) 2
c) 2.5
d) 4

Continuous Probability Models (4 of 4)

A random phenomenon results in a number in the range of 0 to 15. The density curve is shown below. How c
is between 2.5 and 5 compare with t
between 5 and 7.5?



a) It is the same.
b) It is smaller.
c) It is greater.
d) There is not enough information to determine the answer.

Random variables

- A random variable is a variable whose value is a numerical outcome of a random phenomenon.
- The probability distribution of a random variable X tells us what values X can take and how to assign probabilities to those values.

- A finite random variable has a finite list of possible outcomes.
- Random variables that can take on any value in an interval, with probabilities given as areas under a density curve, are called continuous.

Random Variables (2 of 6)

Suppose X is the time required to run a marathon. Would the random variable X be discrete or continuous?

a) continuous

b) discrete

Random Variables (3 of 6)

Suppose X is the number of fans in a football stadium. Would the random variable X be discrete or continuous?

a) continuous

b) discrete

Random Variables (4 of 6)

The probability distribution of a random variable gives __________.

a) the values that the random variable can take

b) the way to assign probabilities

c) both the values that the random variable can take and the way to assign probabilities

d) neither the values that the random variable can take and the way to assign probabilities

Random Variables (5 of 6)

Suppose X is the distance a car could drive with only one gallon of gas. Would the random variable X be discrete or continuous?

a) continuous

b) discrete

Random Variables (6 of 6)

A random variable X and its distribution can be _________ or _________. The distribution of a _________ random variable with finitely many possible values gives the probability of each value. A _________ random variable takes all values in some interval of numbers.

a) discrete; continuous; continuous; discrete

b) discrete; continuous; discrete; discrete

c) discrete; continuous; discrete; continuous

d) None of the answer options is correct.

Personal probability

- A personal probability of an outcome is a number between 0 and 1 that expresses an individual's judgment of how likely the outcome is.
- To be legitimate, a personal probability must obey Rules 1 to 4 of probability.
- It may not match another individual's personal probability of an event.

Personal Probabilities

- Examples:
  - probability that an experimental (never performed) surgery will be successful
  - probability that the defendant is guilty in a court case
  - probability that you will receive an ‘A’ in this course
  - probability that your favorite baseball team will win the World Series in 2020