HW #1 – MATH 6320
SPRING 2015

DUE: TUESDAY JANUARY 27TH

(1) Determine the splitting field of $x^4 + x^2 + 1$ over $\mathbb{Q}$. Also compute its degree.
(2) Determine the splitting field of $x^6 - 4$ over $\mathbb{Q}$. Also compute its degree.
(3) Find all irreducible polynomials of degrees $1, 2$ and 4 over $\mathbb{F}_2$. Then prove their product is $x^{16} - x$.
(4) For any prime p and any nonzero $a \in \mathbb{F}_p$, prove that $x^p - x + a$ is irreducible and separable over $\mathbb{F}_p$. (Suppose α is a root, show that $\alpha + 1$ is also a root).
(5) Let $a > 1$ be an integer and let $n, d \geq 1$ be more integers. Show that $d|n$ if and only if $a^d - 1$ divides $a^n - 1$. Conclude that $\mathbb{F}_{p^d} \subseteq \mathbb{F}_{p^n}$ if and only if d divides n.
(6) Let K be a finite extension of $\mathbb{Q}$. Prove that there are only finitely many roots of unity in K.
(7) Use the fact that $\alpha = 2\cos(2\pi/5)$ is a root of the polynomial $x^2 + x - 1$ to show that the regular 5-gon is constructible by straightedge and compass.