HW #9 – MATH 6310
FALL 2017

DUE: FRIDAY, DECEMBER 1ST

(1) Determine the number of non-isomorphic abelian groups of order 360.
(2) Determine, up to similarity, all real 5×5 matrices A with minimal polynomial[1] $(x-1)^2(x+1)$.
(3) Consider the matrix
$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & -2 & 0 & 1 \\ -2 & 0 & -1 & -2 \end{pmatrix}$$
Find the rational and Jordan canonical forms for A.
(3) Show that any matrix is similar to its transpose.
(4) Let M be a 3×3 complex matrix with $M^6 = M^4$ and $M^4 + M^2 = 2M^3$. Determine the possible Jordan canonical forms for M.
(5) Suppose that $R = \mathbb{Z}[i]$ and M and N are finitely generated R-modules such that
$$M \oplus R^{\oplus 2} \oplus R/2R \cong N \oplus R \oplus R/(1+i)R \oplus R \oplus R/(1-i)R.$$
Is it true that $M \cong N$? (Be careful about which elements are relatively prime).
(6) Let k be a finite field with p elements (and p is prime). Determine the number of 2×2 nilpotent[2] matrices.
(7) Let M be the cokernel of the map $\mathbb{Z}^{\oplus 2} \longrightarrow \mathbb{Z}^{\oplus 3}$ given by the matrix
$$\begin{pmatrix} 3 & 6 \\ 4 & 10 \\ 10 & 22 \end{pmatrix}$$
Write M as a direct sum of cyclic groups.

[1] The lowest degree monic polynomial f such that $f(A) = 0$.
[2] Matrices A such that $A^n = 0$ for some n.