HW #6 – MATH 6310
FALL 2017

DUE: FRIDAY, NOVEMBER 3RD

(1) Suppose that M is a commutative monoid with cancellation. Show that any prime is irreducible.

(2) Show that the monoid under multiplication $\mathbb{Z}[\sqrt{-5}] \setminus \{0\}$ satisfies the following condition:

M has no infinite sequence of elements $a_1, a_2, \ldots$ such that a_{i+1} is a proper factor of a_i.

(3) Show that $\mathbb{Z}[\sqrt{10}]$ is not factorial.

(4*) Let D be a Euclidean domain whose function δ satisfies

(i) $\delta(ab) = \delta(a)\delta(b)$ and

(ii) $\delta(a + b) \leq \max(\delta(a), \delta(b))$.

Show that either D is a field or $D \cong k[x]$ where k is a field.

Hint: Consider the set k of elements $a \in D$ such that $\delta(a) \leq \delta(1) = 1$. Show that k is closed under subtraction and multiplication, and has inverses and so is a field. If $K = D$ you are done otherwise find an element $x \in D \setminus K$ with minimum $\delta(x)$. Now do division and build your polynomials.