

HW #4 – MATH 6310
FALL 2017

DUE: FRIDAY, OCTOBER 6TH

- (1) Show there is no simple group of order 108.
- (2) Let G be a finite group of order p^n , where p is prime and $n \geq 1$. Suppose G acts on a finite set S . Let S' be the subset of S consisting of elements fixed by G :

$$S' = \{x \in S \mid gx = x \text{ for all } g \in G\}.$$

Prove that the order of S' is congruent to the order of S modulo p .

- (3) Show there is no simple group of order 132.
- (4) Let G be a finite p -group for some prime integer $p > 0$. Suppose that $1 \in G$ is the identity and $Z = Z(G) \subseteq G$ is the center. If $N \neq \{1\}$ is normal, prove that $N \cap Z \neq \{1\}$.
- (5) Show there is no simple group of order 56.
- (6) Show that the dihedral group D_n is isomorphic to a semi-direct product of a cyclic of order n by a cyclic group of order 2.