HW #2 – MATH 6310
FALL 2017

DUE: FRIDAY, SEPTEMBER 8TH

(1) Recall that the *index* of a subgroup $H \leq G$ is $[G : H]$, the number of cosets (left = right) of H. Show that every subgroup of index 2 is normal. Conclude that A_n is normal in S_n.

(2) Suppose that H and H' are simple groups and set $G = H \times H'$ a group with the product (componentwise) group structure. Show that every normal subgroup of G that is not $\{1\}$ or G,is isomorphic to H or H'.

Hint: Show that the intersection of normal subgroups is normal.

(3) Compute the cosets of $H = \langle (12) \rangle$ in S_3. Show explicitly with an example that addition of cosets via the formula $(aH)(bH) = (ab)H$ is *not* well defined.

(4) Suppose G is a group. Consider the function from G to itself, $a \mapsto a^{-1}$. Show that this function is an automorphism if and only if G is Abelian.

(5) Determine $\mathrm{Aut} S_3$.

(6) For any $a \in G$ consider the map $\phi_a : G \longrightarrow G$ defined by $\phi_a(x) = axa^{-1}$.
   (i) Show that ϕ_a is an automorphism. (It is called an *inner automorphism*).
   (ii) Show that $a \mapsto \phi_a$ gives us a homomorphism $G \longrightarrow \mathrm{Aut}(G)$ with kernel equal to $Z(G)$ the center[1] of G.
   (iii) Let $\mathrm{Inn}(G)$ be the image of the map in (ii). Show that $\mathrm{Inn}(G) \cong G/Z(G)$.
   (iv) Finally show that $\mathrm{Inn}(G)$ is a normal subgroup of $\mathrm{Aut}(G)$.

[1]Those elements of G that commute with every other element of G