WORKSHEET #5 – MATH 6130
FALL 2018

DUE MONDAY, OCTOBER 15TH

You may work in groups of up to 3. Only one worksheet needs to be turned in per group.

1. For any two points $p = (a_0 : a_1 : \cdots : a_n), q = (b_0 : b_1 : \cdots : b_n) \in \mathbb{P}^n$, show that there is a unique projective line L through those two points and compute the ideal $I(L)$ (based on the above coordinates).

2. Consider $H \subseteq \mathbb{P}^n$ defined by $Z(x_n)$. For a point $(a_0 : a_1 : \cdots : a_n = 1) = p \in \mathbb{P}^n \setminus H$, we can create a map

$$U = \mathbb{P}^n \setminus \{p\} \xrightarrow{\phi_p} H \cong \mathbb{P}^{n-1}$$

which takes $q \in U$, considers the line L through p and q, and takes q to the point $H \cap L$. Write down formulas for the coordinates of this map and verify it is regular on U.

4. Suppose that E is a d-dimensional linear subspace of $\mathbb{P}^n$. Let $L_1, \ldots, L_{n-d}$ be linear forms in $k[x_0, \ldots, x_n]$ which define E. We consider the rational map

$$\phi : P^n \xrightarrow{(L_1:\cdots:L_n)} \mathbb{P}^{n-d-1}.$$

Choose a linear subspace F of $\mathbb{P}^n$ of dimension $n-d-1$ which is disjoint from E. (Simply extend $L_1, \ldots, L_{n-d}$ to a k-basis of the 1-degree elements of $S(\mathbb{P}^n)$, say $L_1, \ldots, L_{n-d}, M_1, \ldots M_{d+1}$ and let those forms define F). For each $p \in \mathbb{P}^n \setminus E$, construct G_p the unique $(d+1)$-dimensional linear subspace of $\mathbb{P}^n$ containing p and E. Show that G_p intersects F in a unique point.

3. With notation as in **2.**, consider $W = Z(x_n, x_{n-1})$ and choose a point $q \in H \setminus W$. Again construct a regular map $V = H \setminus W \xrightarrow{\phi_q} W$ as in **2.** Now, compute the natural domain of the rational map

$$\phi_q \circ \phi_p$$

and describe the map geometrically as a projection from a linear subspace in the sense of the previous problem.