WORKSHEET #2 – MATH 6130
FALL 2018

DUE FRIDAY, SEPTEMBER 7TH

You may work in groups of up to 3. Only one worksheet needs to be turned in per group.

1. Suppose that R is a ring. An nonzero and nonunit element $r \in R$ is called *irreducible* if $r = ab$ implies that either a or b is a unit. Find a affine variety X an irreducible element $r \in k[X]$ such that $Z_X(r)$ is *not* irreducible.

2. Let $X \subseteq \mathbb{A}^3$ be the algebraic set defined by the ideal $I = (x^2 - yz, xz - x)$. Show that X is the union of three irreducible components $X = X_1 \cup X_2 \cup X_3$. Compute $I(X_1), I(X_2)$ and $I(X_3) \subseteq k[x, y, z] = k[\mathbb{A}^3]$.

2. Suppose that $U \subseteq \mathbb{A}^n$ is an open set (in the Zariski topology). Show that U is quasi-compact (every open cover has a finite subcover).

Hint: Convert the open cover into a statement about ideals.

4. Define a regular map $\phi : \mathbb{A}^1 \longrightarrow \mathbb{A}^3$ defined by $\phi(t) = (t^2, t^3, t^4)$. Show that the image of ϕ is closed with $X = \phi(\mathbb{A}^1)$ and find a finite set of generators of the ideal $I(X)$. Is ϕ an injection? Is it an isomorphism onto its image?

I'll ask a group to present this next Friday.

5. Define a regular map $\phi : \mathbb{A}^1 \longrightarrow \mathbb{A}^3$ defined by $\phi(t) = (t^3, t^4, t^5)$. Show that the image of ϕ is closed with $X = \phi(\mathbb{A}^1)$. Find a finite set of generators of the ideal $I(X)$.

Hint: The only way I know how to prove that the "not really obvious" set of elements really are generators is to weight the variables of $\mathbb{A}^3$ appropriately and analyze the heck out of the situation. An extensive hint is provided for this problem in exercise 2.76 in the text.