WORKSHEET #1 – MATH 6130
FALL 2018

DUE FRIDAY, AUGUST 31ST

You may work in groups of up to 3. Only one worksheet needs to be turned in per group.
Recall the following results that we started the class with.

Theorem. *Suppose that $\phi : X \longrightarrow \mathbb{A}^m$ is a regular map. Then $\overline{\phi(X)} = Z(\ker \phi^*)$.*

Theorem. *A regular map $\phi : X \longrightarrow Y$ is an isomorphism if and only if $\phi^* : k[Y] \longrightarrow k[X]$ is an isomorphism.*

1. Show that a regular map $\phi : X \longrightarrow Y$ is dominant if and only if ϕ^* injects.

2. Show that $\phi : X \longrightarrow Y$ induces an isomorphism of X onto a closed subset of Y if and only if ϕ^* surjects.

3. Let $X = Z(y - x^2) \subseteq \mathbb{A}^2$. Show that X is a variety isomorphic to $\mathbb{A}^1$. Then show that $Y = Z(xy - 1)$ is not isomorphic to $\mathbb{A}^1$.

4. Let $X = Z(y^2 - x^3) \subseteq \mathbb{A}^2$. Consider the regular map $\phi : \mathbb{A}^1 \longrightarrow X$ defined by $\phi(t) = (t^3, t^2)$. Show that ϕ is a bijection but not an isomorphism.

5. Let $f \in k[x, y]$ be an irreducible quadratic polynomial. If $\mathrm{char} k \neq 2$, show that either $Z(f)$ is isomorphic to $\mathbb{A}^1$ or isomorphic to $Z(xy - 1)$.

6. Suppose that X is an irreducible topological space. If $U \subseteq X$ is a non-empty open set, show that U is also irreducible. Further show that if U_1 and U_2 are two non-emtpy sets, that $U_1 \cap U_2$ is non-empty.