WORKSHEET #2 – MATH 5405
SPRING 2016

DUE: TUESDAY, 1/26/2016

We will play around with finite fields that are *not* prime fields. Only one worksheet is required per group.

1. Suppose that F is a finite field of characteristic p. Prove that F has p^n elements for some integer n.

Hint: First note that F contains a copy of $\mathbb{Z}/p$. Deduce that F is a finite dimensional $\mathbb{Z}/p$ vector space. Choose a basis and then just count the elements.

2. Consider the ring of polynomials $\mathbb{Z}/3[x]$ and consider the polynomial $f(x) = x^2 + 1$. Show that $f(x)$ is irreducible. Conclude that $\mathbb{Z}/3[x]/(x^2 + 1)$ is a field.

3. How many elements does the field $\mathbb{Z}/3[x]/(x^2+1)$ have?

4. Write down a multiplication table for $\mathbb{Z}/3[x]/(x^2+1)$.

5. Which elements of the field $\mathbb{Z}/3[x]/(x^2+1)$ are primitive roots?