MACAULAY2 HOMEWORK

DUE FRIDAY, SEPTEMBER 27TH

1. Use the command `isPrime` to determine which of the following ideals are prime.

(i) $I = \langle x^2 + y, y^2 + z, z^2 + x \rangle \subseteq \mathbb{Q}[x, y, z]$.
(ii) $J = \langle x + y, y^2 + z^3 \rangle \subseteq \mathbb{Q}[x, y, z]$
(iii) $K = \langle x + y^2, y^2 + z^3 \rangle \subseteq \mathbb{Q}[x, y, z]$
(iv) $I * J + K$
(v) $J + K \cap (I + J)$

2. Let $R = \mathbb{Q}[x, y]$ and map between two free modules $\phi : R^3 \longrightarrow R^2$ such that the cokernel is not a free module (and is nonzero). You can use Macaulay2 to help. You can try (in vain) to find an example where the kernel is not free, but you'd need more variables for that. For a challenge, find an example the kernel is non-free for a polynomial ring with more variables.

3. Let $\phi : R^3 \longrightarrow R^2$ be the map you constructed in 2. Fix $M = \langle x^2 + y^3 \rangle$. Find presentations of the kernel and cokernel of the induced maps

$$\mathrm{Hom}_R(M, R^3) \longrightarrow \mathrm{Hom}_R(M, R^2)$$

and

$$M \otimes R^3 \longrightarrow M \otimes R^2.$$

Hint: You'll have to figure out how to apply Hom and $\otimes$ to maps of modules in Macaulay2, it isn't as hard as you might think.

4. Look up the function `radical` in Macaulay2. For the ideals you constructed in (i)–(v) in problem 1., compute the radical of each of the non-prime ideals (the prime ones were already radical of course). Were any of the non-prime ideals radical?

5. Consider the subring

$$S = \{ f \in \mathbb{Q}[x] \mid f(0) = f(1) = f(2) \} \subseteq \mathbb{Q}[x]$$

Find a presentation for S (ie, write $S = \mathbb{Q}[a, b, c, \ldots]/I$, find out how many variables $a, b, \ldots$ you need and what the ideal I is, Macaulay2 can help with the last part, and perhaps the first part if you are clever).