HOMEWORK # 7
DUE FRIDAY DECEMBER 9TH

MATH 538 FALL 2011

1. Suppose that A is a ring and that M and N are A-modules. A module L together with a short exact sequence $0 \to M \to L \to N \to 0$ is called an *extension of* M *and* N. For example, $M \oplus N$ is an extension of M and N with the usual short exact sequence (it is called the *trivial extensions*). We say that two extensions L and L' are equivalent if there is a commutative diagram:

$$\begin{array}{ccccccccc} 0 & \longrightarrow & M & \longrightarrow & L & \longrightarrow & N & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle \mathrm{id}} & & \downarrow{\scriptstyle \sim} & & \downarrow{\scriptstyle \mathrm{id}} & & \\ 0 & \longrightarrow & M & \longrightarrow & L' & \longrightarrow & N & \longrightarrow & 0 \end{array}$$

Prove that there is a bijective correspondence between equivalence classes of extensions and elements of $\mathrm{Ext}^1(N, M)$. Additionally, prove that under this correspondence, the element $0 \in \mathrm{Ext}^1(N, M)$ corresponds to the trivial extension.

2. Let $R = k[x, y, z]$ where k is a field. Prove that $x, y(1-x), z(1-x)$ is a regular sequence on R but $y(1-x), z(1-x), x$ is not a regular sequence on R.

3. Suppose that $x_1, \dots, x_t \in A$ is a regular sequence on a module M. Prove that $\mathrm{Tor}_1^A(M, A/\langle x_1, \dots, x_t \rangle) = 0$.

4. Prove that the subalgebra $S = k[u^4, u^3v, uv^3, v^4] \subseteq k[u, v]$ is not Cohen-Macaulay but that $k[u^4, u^3v, u^2v^2, uv^3, v^4]$ is Cohen-Macaulay.