

INFO FOR THE FINAL EXAM

MATH 538 FALL 2011

The final exam will be oral, individual and in a seminar room. The exam will be 20-30 minutes. You can schedule your exam for any time after thanksgiving including during finals week. Send me an email saying when you'd like to schedule it.

The oral final may include questions on the following topics:

The geometry of Spec: In particular, what are irreducible components, and what do they correspond to in the ring. How do you detect different connected components. You should be able to prove that maps of rings induce continuous maps on Spec and you should be able to explain what modding out by an ideal does. Finally, you should be able to state the Nullstellensatz.

Localization: You should be able to say intelligent things about how one proves that localization is exact and how this relates to tensor products. You should also be familiar with what localization does with respect to Spec.

Noetherian and Artinian rings: You should be familiar with the definitions. You should also familiarize yourself with the proof that all Artinian rings are Noetherian (and have finite dimension). You should be aware of why working with finitely generated modules (and their submodules) is nicer for Noetherian rings.

Modules and short exact sequence: You should be able to define exact and short exact sequences of modules as well as produce examples and non-examples. You should be able to explain the functors Hom and tensor, and what their exactness properties are. You should be able to do simple diagram chases / proving that things are exact.

Primary decomposition: You should be able to describe the existence and uniqueness theorems for primary decomposition. You should also be able to give examples of ideals that are primary, but not prime, and also non-primary ideals.

Integral extensions: You should be able to define and give examples of integral and finite extensions. You should know examples and non-examples of these extensions. You should be able to state and explain the going-up and going-down theorems. Finally, you should be able to state Noether normalization.

Completion: You should be able to talk intelligently about completion of ideals. You should also be able to explain exactness properties and be able to explain exactly why the Artin-Rees lemma is relevant.

Dimension: You should be able to discuss dimension of rings as well as some of the more recent discussion of different characterizations of dimension of local rings. You should be able to identify dimensions in some easy examples.