WORKSHEET #3 – MATH 536
SPRING

DUE MONDAY, FEBRUARY 24TH

Let H and K be groups and let $\phi : K \longrightarrow \mathrm{Aut}(H)$ be a group homomorphism. We get a left action of K on H by $k.h = (\phi(k))(h)$. Let

$$G = H \times K$$

with the following binary operation:

$$(h_1, k_1)(h_2, k_2) = (h_1(k_1.h_2), k_1k_2).$$

This is called the *semi-direct product of* K *and* H, and is denoted by $H \rtimes K$.

1. Show that $G = H \rtimes K$ is a group.

2. If we identify H with $\{(h, 1)\}$, then show that H is a *normal* subgroup of G. This helps explain the notation, the H is the normal factor in $H \rtimes K$.

3. Let $H = \mathbb{Z}/4\mathbb{Z}$ and $K = \mathbb{Z}/2\mathbb{Z}$. Let the map $\phi : K \longrightarrow \mathrm{Aut}(H)$ be the map which sends $[1]$ to the inversion bijection. In other words, $\phi([1])(k) = -k$ and $\phi([0])(k) = k$. Show that $H \rtimes K$ is isomorphic D_8 (the dihedral group on the square, which other books will denote by D_4).

4. Suppose G is a group with K an subgroup of G and H a normal subgroup of G. Further suppose that $HK = G$ and that $H \cap K = \{1\}$. Let $\phi : K \longrightarrow \mathrm{Aut}(H)$ be the map which conjugates by k. In other words $\phi(k)(h) = khk^{-1}$. Show that $G \cong H \rtimes K$.

Hint: First show that every element of HK can be written uniquely as hk for some $h \in H$ and $k \in K$. Then define a map $HK \longrightarrow H \rtimes K$ sending $hk \mapsto (h, k)$. You need to show that this is a homomorphism (it is obviously a bijection).