WORKSHEET #2

The following 8 transformations make up the group D4.

You may assume that D4 is indeed a group (for now...).

1. Prove that D4 is *not* Abelian.

Solution: $f1 \cdot f2 =$ "do f2 first then f1" $= r90$ but $f2 \cdot f1 =$ "do f1 first then f2" $= r270$.

3. Find the cyclic subgroups generated by:

(a) e
(b) $r90$,
(c) $f3$.

Solution:

(a) $\{e\}$
(b) $\{r90, r180, r270, e\}$,
(c) $\{f3, e\}$.

4. Identify the smallest subgroup of D4 which contains both $r180$ and $f1$.

Hint: It must contain $r180 \cdot f1$ and $f1 \cdot r180$ and $f1 \cdot f1$ and ... and inverses to all these elements.

Solution: $r180 \cdot f1 = f3$, $f1 \cdot r180 = f3$, $f1 \cdot f1 = f3 \cdot f3 = r180 \cdot r180 = e$. Also, $f3 \cdot f1 = r180 = f1 \cdot f3$ and $r180 \cdot f3 = f1 = f3 \cdot r180$. These combine to show that the set $\{e, f1, f3, r180\}$ are closed under multiplication (checking closure with respect to multiplication by e isn't necessary for obvious reasons). It is also associative since the multiplication of D4 is associative. The identity is included in the set and certainly inverses exist (each element is its own inverse)

Alternately, this is a finite subset of a group which is closed under the group operation, and so it is a subgroup by a theorem from in class / from the book.

5. Write down a complete group multiplication table for D4. I will interpret the entry in the $r90$ row and $f1$ column to be $r90 \cdot f1$. What do you notice about every column and row?

Hint: You are working in groups. Do one row and column together, and then divide the labor.

	e	$r90$	$r180$	$r270$	$f1$	$f2$	$f3$	$f4$	
e	e	$r90$	$r180$	$r270$	$f1$	$f2$	$f3$	$f4$	
$r90$	$r90$	$r180$	$r270$	e	$f4$	$f1$	$f2$	$f3$	
$r180$	$r180$	$r270$	e	$r90$	$f3$	$f4$	$f1$	$f2$	
$r270$	$r270$	e	$r90$	$r180$	$f2$	$f3$	$f4$	$f1$	
$f1$	$f1$	$f2$	$f3$	$f4$	e	$r90$	$r180$	$r270$	
$f2$	$f2$	$f3$	$f4$	$f1$	$r270$	e	$r90$	$r180$	
$f3$	$f3$	$f4$	$f1$	$f2$	$r180$	$r270$	e	$r90$	
$f4$	$f4$	$f1$	$f2$	$f3$	$r90$	$r180$	$r270$	e	

I notice that in each row and column, every element of the group appears exactly once.

6. Use your table to deduce that D4 has inverses.

Solution: Indeed, I can see this by inspection, most elements are their own inverses except for $r90$ and $r270$ which are each others inverses.