SYLLABUS – MATH 435

BASIC ABSTRACT ALGEBRA

Description: This course will introduce groups, rings and fields, the fundamental objects of study in what is known as *modern (or abstract) algebra*. Modern algebra has applications to many fields, particularly theoretical physics, cryptography and coding theory – some of which we will discuss in class. If taught properly, it should also be the most challenging class for undergraduate math majors.

- **Time:** Monday, Wednesday, Friday 3:35pm – 04:25pm
- **Location:** 307 Boucke
- **Instructor:** Karl Schwede
- **Contact information:**
  - email: schwede@math.psu.edu
  - office: McAllister, 318C
  - office phone: (814)865-8439
  - website: `http://www.math.psu.edu/schwede/math435`
- **Office hours:** Monday 1:30 – 2:30, Thursday 10:00 – 11:00, Friday 2:15 – 3:15.
- **Textbooks:**
  - Primary Text: "Abstract Algebra", (3rd Edition), *I. N. Herstein*
  - Suggested alternate Text: "Contemporary Abstract Algebra", *Joseph Gallian*

Grade: Your grade will be determined by the following formula.

20% Exam #1, tentatively Wednesday, February 22nd
20% Exam #2, tentatively Friday, March 30th
30% Homework/Quizzes/Worksheets, homework will be assigned and collected approximately weekly. Your lowest homework grade will be dropped.
30% Final Exam, date TBA

Generally speaking, late homework will not be accepted and missed exams and quizzes cannot be made-up. In unavoidable circumstances, you must speak with the instructor *prior* to missing the homework/exam/quiz in order to receive credit. In such cases, the impact on the grade will be dealt with on a case by case basis.

Students are allowed, and even encouraged to work together when solving homework problems. However, each student must turn in his or her own write-up.

Prerequisites: Students taking this course should be familiar with and comfortable using basic proof strategies such as induction, proof by contradiction. Students should also be comfortable with the basic formalisms of mathematics such as sets, functions, relations, etc. You will also need to be familiar with basic properties of matrices (multiplying matrices, inverses, determinants, etc). We will do a crash course on these topics during the first week. The official prerequisite is Math 311W or equivalent, please talk to me if you have not had one of these courses.

Academic Integrity: All Penn State policies regarding ethics and honorable behavior apply to this course.

Warm-up homework set: Due Friday, January 13th

(1) What is wrong with the following inductive proof that either all cats are the same color? For example, will will "prove" they are all orange, or all black, or all grey, or etc. We will ignore the possibility of a multi-colored cat.

> "We will show that for any set S of n cats, all cats in S are all the same color. For the base case, we consider a set S_1 with 1 cat in it. Clearly that cat has the same color as itself and so the base case is proven. Now suppose that S is a set of $n+1$ cats (we label these cats C_0 to C_n). Suppose that $A \subset S$ is the set $\{C_0, \dots, C_{n-1}\}$ and $B \subset S$ is the set $\{C_1, \dots, C_n\}$. By our inductive hypothesis, all the cats in the set A are all the same color, likewise with the cats in B. Now observe that $C_1, \dots, C_{n-1}$ are in both sets and so C_0 has the same color as $C_1, \dots, C_{n-1}$ (since they are all in A) which has the same color as C_n (since they are all in B). Thus all cats in S have the same color. This completes the proof by induction."

(2) Find a formula for $3 + 5 + 7 + \cdots + (2n - 1)$ and use inductive reasoning to prove that your formula is correct.

Recall that a function is called *injective* if it is one-to-one and that a function is called *surjective* if it is onto.

(3) Suppose that $f : S \to T$ and $g : T \to U$ are two functions and consider the composition $g \circ f : S \to U$.
   (a) Suppose that $g \circ f$ is surjective, prove that g is also surjective.
   (b) Suppose that g and f are both injective, prove that $g \circ f$ is also injective.
   (c) Give an example of two functions g and f such that g is not injective, f is injective, but $g \circ f$ is injective.

(4) Consider the following proof that there are infinitely many prime natural numbers.

> "Suppose that there were finitely many primes, $p_1, \dots, p_n$. Consider the new number $m = p_1 \cdot p_2 \dots p_n + 1$. It is clear that $m > p_i$ for $i = 1, \dots, n$ and so m is not prime. But $p_i \not| m$ for each $i = 1, \dots, n$ since $m = p_i(\prod_{j \neq i} p_i) + 1$. Now, every integer m is a product of primes by the fundamental theorem of arithmetic, but no prime divides m, a contradiction.

The proof is correct, but consider the following question inspired by it. If we set $p_1, \dots, p_n$ to be the first n primes, and define $m = p_1 \cdot p_2 \dots p_n + 1$, is it true that m is always a prime number?

Either prove that this is correct or provide a counter-example (it is ok to use a calculator).

(5) Prove that the square root of 15 is irrational
Hint: Suppose $\sqrt{15} = a/b$ for some positive $a, b \in \mathbb{Z}$, square both sides and derive a contradiction using unique factorization.

(6) Prove directly that if A and B are 2×2 matrices, then $\det(AB) = \det(A) \cdot \det(B)$.

(7) Find the inverse of the square matrix below or prove it is not invertible:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{bmatrix}$$