INFO ON THE FINAL EXAM

MATH 435 SPRING 2012

There will be 6 pages of regular questions on the exam and one extra-credit question.

(1) There will be two pages of short answer questions (for example, define the term *group,ring,field, group action*, give an example of a field with 4 elements, prove that a subring of an integral domain is an integral domain, show that various polynomials are irreducible, or compute $[K : F]$ for some field $F \subseteq K$, prove a map is a ring homomorphism).
(2) There will be one proof problem focusing on a specific field and some extensions of it (and possibly polynomials over that field, etc.)
(3) There will be one abstract proof-based problem focusing on new material.
(4) I will ask you two of the following:
   (a) State and prove Lagrange's theorem.
   (b) Prove the first isomorphism theorem for groups.
   (c) Prove the orbit-stabilizer theorem.
   (d) Prove Cauchy's theorem for Abelian groups (in other words, if A is an Abelian group and a prime p divides the order of A, then A contains an element of order p).
   (e) Prove that $[L : F] = [L : K] \cdot [K : F]$ if $F \subseteq K \subseteq L$ are field extensions.
   (f) Suppose that F is a field and $K \supseteq F$ is an extension field. Set $E(K)$ to be the set of elements in K that are algebraic over F. Prove that $E(K)$ is a field (in particular, that it has closure under $+$ and $\cdot$).
   (g) Suppose that $a \in K \supseteq F$ is algebraic over F. Prove that $F[a] \simeq F[x]/\langle f(x)\rangle$ where $f(x) \in F[x]$ is the minimal polynomial for a over f.
   (h) Prove that $k[x]$ is a principal ideal domain if k is a field.
   (i) Prove that every finite extension field is algebraic.

The extra credit problem will be related to compass and straight-edge constructions.