EXTRA CREDIT OVER SPRING BREAK #1

DUE MONDAY MARCH 12TH

Suppose that G_3 is the Rubik's cube group, as described on worksheet #5.

Exercise 0.1. Suppose that G_3 acts on the 12 unoriented edge cubies. Explicitly find the orbit of each edge cubie. Justify your answer. (1 point)

Here is a simple exercise that has nothing to do with what we've done so far.

Exercise 0.2. Suppose that G is acting on a set S (on the right, just for a change) and that $T \subseteq S$ is a finite subset. Consider now the subset of G

$$\mathrm{Inv}_G(T) = \{g \in G \,|\, x.g \in T, \text{ for all } x \in T \text{ and } g \in G\}$$

Prove that $\mathrm{Inv}_G(T)$ is a subgroup of G. (1 point)

To do the rest of the extra credit, you will need to read parts of a set of notes written by Janet Chen and linked from our course homepage:

http://www.math.psu.edu/schwede/math435/index.html

Now, we ask the following questions. We consider G_3 acting on on all of the possible configurations of the Rubik's cube, in other words on the set S of size $8!3^8 12!2^1 2$ which we discussed in class on Friday.

Exercise 0.3. Suppose that x is the oriented upper-right-front (urf) corner cubie. Consider G_3 acting on the cubies. Find $|\mathrm{Stab}_{G_3}(x)|$. (1 point)

Exercise 0.4. Consider the following sets:

(a) A the set of all *valid* Rubik's cube configurations that are solved except for the top layer. In other words, Rubik's cubes that are studied in Worksheet #6.

(b) B the set of all *valid* Rubik's cube configurations that are solved except for the 4 top edge cubies, which are all properly oriented, but are in the wrong position.

Find the size of A and the size of B. (1 point each)

Exercise 0.5. We continue to consider the sets A and B from a previous exercise. Compute the order of $\mathrm{Inv}_G(A)$ and of $\mathrm{Inv}_G(B)$. (2 points)