QUIZ # 2

MATH 435 SPRING 2011

1. Compute all of the left cosets of the subgroup $\{1, 10\}$ inside the group $U(11)$ (the group of integers between 1 and 11 relatively prime to 11, under multiplication). Is $\{1, 10\}$ a normal subgroup? (2 points)

Solution : First let me verify that $\{1, 10\}$ is indeed a subgroup, and in particular check closure. The only thing of any difficulty here is that $10^2 = 100 \equiv_{\text{mod } 11} = 1$, which indeed shows it is closed and also a subgroup.

The cosets are $\{1, 10\}, \{2, 9\}, \{3, 8\}, \{4, 7\}$ and $\{5, 6\}$ as one can readily compute. The subgroup $\{1, 10\}$ is normal because the ambient group $U(11)$ is Abelian.

2. Suppose that $\phi : A \to B$ is a group homomorphism. Further suppose that $H \subseteq A$ is a subgroup and that $a \in A$ is an element. Suppose that for every element $x \in aH$, $\phi(x) = \phi(a)$ (in other words, ϕ has the same output no matter what element of the coset aH is plugged into it). Prove that $H \subseteq \ker \phi$. (3 points)

Solution: Fix some $h \in H$, then $ah \in aH$, so $\phi(ah) = \phi(a)$ by hypothesis. But because ϕ is a homomorphism, $\phi(ah) = \phi(a)\phi(h)$ so that $\phi(a)\phi(h) = \phi(a)$. Thus $\phi(h) = e_B$ which implies that $h \in \ker \phi$ and thus that $H \subseteq \ker \phi$.