QUIZ # 1

MATH 435 SPRING 2011

1. Consider the group $\mathbb{Z}_{\text{ mod } 15}$ under addition. Write down all the elements of the group and identify the order of each element. For which $g \in \mathbb{Z}_{\text{ mod } 15}$ is it true that $\mathbb{Z}_{\text{ mod } 15} = \langle g \rangle$? (3 points)

Solution: The elements of $\mathbb{Z}_{\text{ mod } 15}$ are $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$.

- The elements of order 1 are: 0.
- The elements of order 3 are: $5, 10$.
- The elements of order 5 are: $3, 6, 9, 12$.
- The elements of order 15 are: $1, 2, 4, 7, 8, 11, 13, 14$.

The elements g such that $\mathbb{Z}_{\text{ mod } 15} = \langle g \rangle$ are exactly the elements of order 15.

2. Prove that S_n is not cyclic for any $n \geq 3$. (2 points)
Hint: One approach is to consider the number of subgroups of order 2 that S_n has. There are many other approaches however.

Solution: S_n is a finite group which has at least two subgroups of order two, for example $\{(12), e\}$ and $\{(13), e\}$ are always subgroups of S_n for $n \geq 3$. On the other hand, every finite cyclic group has at most one subgroup of order 2 (it has at most one subgroup of any given order). Thus S_n can't possibly be cyclic.