# HOMEWORK # 4
# DUE FRIDAY FEB. 4TH

MATH 435 SPRING 2011

**1.** Prove that every infinite group contains infinitely many different subgroups.
**2.** Suppose that $H$ and $K$ are subgroups of a group $G$. Prove that $H \cap K$ is also a subgroup of $G$.
**3.** Give an example of a group $G$ with two subgroups $H, K \subseteq G$ such that $H \cup K$ is also a subgroup.
**4.** Suppose that $G$ is an abelian group and that $H, K \subseteq G$ are two subgroups. Consider the set

$$HK = \{hk | h \in H, k \in K\}.$$

Prove that $HK$ is a subgroup.
**5.** Suppose that $G = \langle a \rangle$ and that $|a| = |G| = 24$. List all generators for the subgroup $H \subseteq G$ of order eight (ie, $|H| = 8$).
**6.** Prove that every Abelian group of order 6 is cyclic.
**7.** Suppose that $G = \langle a \rangle$ is cyclic and that $|G| = 24$. Further suppose that $b \in G$ is an element such that $b^8 \neq e$ and $b^{12} \neq e$. Prove that $G = \langle b \rangle$.
**8.** Suppose that $a, b \in G$ where $G$ is a group. Suppose that $|a| = 12$ and $|b| = 22$, and $\langle a \rangle \cap \langle b \rangle \neq \{e\}$. Show that $a^6 = b^{11}$.
**9.** Consider the set $\{4, 8, 12, 16\}$. Show that this set is a cyclic group under multiplication modulo 20.
**10.** Prove that $U(2^n)$ is not cyclic for all $n \geq 3$.

**Extra credit (worth up to 3 points to your homework score):**
(Due Wednesday, January 9th)

Recall that $U(n)$ is the group of positive integers both less than and relatively prime to $n$, under multiplication modulo $n$. Compute many examples of $U(n)$ and try to determine for which $n$, $U(n)$ is cyclic. Write down a conjectural formula with substnantial justification. If you can prove your formula is correct, or parts of it are, that's even better (and will be worth additional points above and beyond what's listed above).

*Hint:* It is ok to work with other up to 3 other people and turn in a joint project. Furthermore, writing a computer program to do the computations is quite reasonable – let me know if you'd like help with this. If you do write some software, please also turn in the source code.