## HOMEWORK \#9 - MATH 3210,

## FALL 2019

DUE TUESDAY, NOVEMBER 5TH

5.2 \#1. Show that if a function $f$ on a bounded interval can be written in the form $g-h$ for functions $g$ and $h$ which are non-decreasing on $[a, b]$, then $f$ is integrable on $[a, b]$.
5.2 \#9. Prove that if $f$ is integrable on $[a, b]$ then so is $f^{2}$.

Hint: See the text for a substantial hint.
5.2 \#10. Prove that if $f$ and $g$ are integrable on $[a, b]$ then so is $f \cdot g$.

Hint: See the text for a substantial hint.
5.3 \#4. Compute

$$
\frac{d}{d x} \int_{1 / x}^{x} e^{-t^{2}} d t .
$$

5.3 \#5. If $f(x)=-1 / x$, then $f^{\prime}(x)=1 / x^{2}$. Thus, Theorem 5.3.1 seems to imply that

$$
\int_{-1}^{1}\left(1 / x^{2}\right) d x=f(1)-f(-1)=-1-1=-2 .
$$

However, $1 / x^{2}$ is a positive function so its integral over $[-1,1]$ should be positive. What is wrong?
5.3 \#6. If $f$ is a differentiable function on $[a, b]$ and $f^{\prime}$ is integrable on $[a, b]$, then find $\int_{a}^{b} f(x) f^{\prime}(x) d x$.

