HOMEWORK #5 – MATH 3210,
FALL 2019

DUE TUESDAY, SEPTEMBER 24TH

2.5, #1. Give an example of a nested sequence of bounded open intervals that does not have a point in its intersection.

2.5, #2. Give an example of a nested sequence of closed but unbounded intervals which does not have a point in its intersection.

2.5, #5. Which of the following sequences has a convergent subsequence. Justify your answer.

(a) $a_n = (-2)^n$.

(b) $a_n = \frac{5+(-1)^n n}{2+3n}$.

(c) $a_n = 2^{((-1)^n)}$.

2.5, #7. For each of the following sequences, determine how many different limits of subsequences there are. Justify your answer.

(a) $\{1 + (-1)^n\}$
(b) $\{\cos(n\pi/3)\}$
(c) $\{1, 1/2, 1, 1/2, 1/3, 1, 1/2, 1/3, 1/4, 1, 1/2, 1/3, 1/4, 1/5, \dots `\}$

2.5, #8. Does the sequence $\sin(n)$ have a convergent subsequence? Why?

2.6, #1. Find the lim sup and lim inf for the following sequences.

(a) $a_n = (-1)^n$
(b) $a_n = (-1/n)^n$
(c) $a_n = \sin(n\pi/3)$

2.6, #6. If $k \geq 0$ and $\limsup a_n$ is finite, prove that

$$\limsup(ka_n) = k \limsup(a_n).$$

2.6, #7. If $a_n \geq 0$ and $b_n \geq 0$, prove that $\limsup(a_n b_n) \leq (\limsup a_n)(\limsup b_n)$.

3.1, #1. If f is a function with domain $[0, 1]$, what is the domain of $f(x^2 - 1)$?

3.1, #4. Show that the function $f(x) = |x|$ is continuous on all of $\mathbb{R}$.