HOMEWORK #4 – MATH 3210, FALL 2019

DUE TUESDAY, SEPTEMBER 17TH

2.3, #1. Use the Main Limit Theorem to find

$$\lim \frac{2n^3 - n + 1}{3n^3 + n^2 + 6}$$

2.3, #6. Prove that a sequence $\{a_n\}$ is both bounded above and bounded below if and only if its sequence of absolute values $\{|a_n|\}$ is bounded above.

2.3, #8. Prove that if $\{b_n\}$ is a sequence of positive terms and $b_n \longrightarrow b > 0$, then there is a number $m > 0$ such that $b_n \geq m$ for all n.

2.3, #9. Prove part (d) of Theorem 2.3.6 from the text.
Hint: Use the previous exercise.

2.3, #10. Prove part (f) of Theorem 2.3.6.

Hint: Use the identity

$$x^k - y^k = (x - y)(x^{k-1} + x^{k-2}y + \cdots + y^{k-1})$$

2.4, #4. Let $\{d_n\}$ be a sequence of 0s and 1s and define a sequence of numbers $\{a_n\}$ by

$$a_n = d_1 2^{-1} d_2 2^{-2} + \cdots + d_n 2^{-n}.$$

Prove that this sequence converges to a number between 0 and 1.

2.4, #5. Let a_n be a series with positive terms. Let s_n be the sequence of partial sums of the a_n, that is:

$$s_n = \sum_{k=1}^{n} a_k.$$

Prove that $\lim s_n = \infty$ or $\lim s_n = L$ for some some $L \in \mathbb{R}$. In particular, either way, the limit exists.

2.4, #9. Prove that $\lim \frac{2^n}{n} = \infty$.