

**HOMEWORK #11 – MATH 3210,  
FALL 2019**

DUE TUESDAY, NOVEMBER 12TH

**6.2, #2 + #3.** Determine whether the indicated series converges. Justify your answer by indicating the test to be used and carrying out the details of that test.

**#2.**

$$\sum_{k=1}^{\infty} \frac{\ln(k)}{k^2}$$

**#3.**

$$\sum_{k=1}^{\infty} \frac{k2^k}{3^k}$$

**6.2, #5 + #7.** Determine whether the indicated series converges. Justify your answer by indicating the test to be used and carrying out the details of that test.

**#5**

$$\sum_{k=1}^{\infty} \frac{k}{(3 + (-1)^k)^k}$$

**#7**

$$\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2 - k + 2}$$

**6.2, #11.** Prove that if  $\sum_{k=1}^{\infty} a_k$  converges absolutely and  $\{b_k\}$  is a bounded sequence, then

$$\sum_{k=1}^{\infty} a_k b_k$$

also converges absolutely.

**6.2, #12.** Prove that if  $\sum_{k=1}^{\infty} a_k$  and  $\sum_{k=1}^{\infty} b_k$  are series and  $a_k = b_k$  except for finitely many terms, then the two series either both converge or both diverge.

$$\sum_{k=1}^{\infty} a_k b_k$$

also converges absolutely.

**6.3, #2 + #4.** Determine whether the following series converges absolutely, converges conditionally, or diverges. Justify your answer.

**#2.**

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}$$

**#4.**

$$\sum_{k=1}^{\infty} (-1)^{k-1} \frac{2^k}{2^k + k^2}$$

**6.3, #6.** Give an example of two convergent series  $\sum_{k=1}^{\infty} a_k$  and  $\sum_{k=1}^{\infty} b_k$  such that the series  $\sum_{k=1}^{\infty} a_k b_k$  diverges.

**6.3, #10.** Modify the proof of Theorem 6.3.4 to show that if  $\sum_{k=1}^{\infty} a_k$  is conditionally convergent, then there exists a rearrangement that converges to  $\infty$ .

**6.3, #11.** The geometric series  $\sum_{k=0}^{\infty} 2^{-k}$  converges to 2. Use Theorem 6.3.6 to show that the series

$$\sum_{k=0}^{\infty} (k+1)2^{-k}$$

converges to 4.



**6.4, #1.** Prove that the function  $\frac{f(x)=\sum_{k=1}^{\infty} x^k}{k^2}$  is continuous on the interval  $[-1, 1]$ .