HOMEWORK #10 – MATH 3210, FALL 2019

DUE TUESDAY, NOVEMBER 12TH

5.3 #2. Find

$$\frac{d}{dx}\int_1^x \cos(1/t)dt \quad \text{for } x > 0$$

5.3 #7. Let f be a continuous function on the interval $[0, 1]$. Express

$$\int_0^{\pi/2} f(\sin(\theta)) \cos(\theta) d\theta$$

as an integral only involving the function f.

5.4 #5. Using Definition 5.4.8 and the properties of exp, prove the laws of exponents.

$$a^{x+y} = a^x a^y \text{ and } a^{xy} = (a^x)^y.$$

5.4 #7. Find an antiderivative for a^x for each $a > 0$.

5.4 #8. Prove Theorem 5.4.9. In other words, show that the inverse function $\log_a(x)$ of a^x satisfies the formula $\log_a(x) = \frac{\ln(x)}{\ln(a)}$.

5.4 #9. For which values of $p > 0$ does the impropoer integral $\int_1^\infty \frac{1}{x^p}\,dx$ converge. Justify your answer.

5.4 #11. Show that

$$\int_{-\infty}^{\infty} \frac{\sin(x)}{1+x^2}$$

converges. Can you tell what it converges to?

6.1 #1. Determine whether the following series converges. Justify your answer.

$$\sum_{k=2}^{\infty} \frac{k-1}{2k+1}$$

6.1 #3. Determine whether the following series converges. Justify your answer.

$$\sum_{k=0}^{\infty} \frac{2^{k+1}}{3^k}$$

6.1 #8. Determine whether the following series converges absolutely. Justify your answer.

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}}.$$

6.1, #12. Consider the decimal expansion $0.d_1d_2d_3d_4\ldots$ of a real number between 0 and 1, where $\{d_k\}$ is a sequence of integers between 0 and 9. This decimal expansion represents the sum of a certain infinite series. What series is it and why does it converge?