WORKSHEET #6 – MATH 3210
FALL 2018

NOT DUE

You may work in groups of up to 4. This worksheet covers similar material to the upcoming midterm.

1. Short answer questions.

(a) Give an example differentiable function whose derivative is not continuous.

(b) Give a precise definition of the expression $\lim_{x \longrightarrow \infty} f(x) = L$.

(c) Suppose $f : (a, b) \longrightarrow \mathbb{R}$ is differentiable at $c \in (a, b)$, give a short proof that f is continuous at c.

(d) Suppose that $f(x) = e^{\sin(x)}$ with domain $(-\pi/2, \pi/2)$. Compute $(f^{-1})'(1)$.

(e) Suppose $f : [a, b] \longrightarrow \mathbb{R}$ is continuous. Precisely define what it means for $c \in [a, b]$ to be a critical point.

2. More short answer questions.

(a) Precisely state the Mean Value Theorem.

(b) Suppose $f : (a,b) \longrightarrow \mathbb{R}$ is differentiable. Use the Mean Value theorem to give a short proof of the fact that if $f'(x) > 0$ for all $x \in (a,b)$, then f is strictly increasing.

(c) Precisely state Cauchy's Mean Value theorem.

(d) Use L'Hôpital's rule to compute $\lim_{x \longrightarrow 1} \frac{\ln(x)}{x^1 - 1}$.

(e) Precisely state what it means for a function $f : [a,b] \longrightarrow \mathbb{R}$ to be Riemann integrable on $[a,b]$.

(f) Suppose that $f : [a,b] \longrightarrow \mathbb{R}$ is a function and that $P \subseteq Q$ are partitions. Order the following from smallest to largest using $\leq$. $U(f,P)$, $U(f,Q)$, $L(f,P)$, $L(f,Q)$.

3. Even more short answer questions.

(a) Give an example of a non-integrable function $f : [0,1] \longrightarrow \mathbb{R}$.

(b) If $f : \mathbb{R} \longrightarrow \mathbb{R}$ is differentiable. Is it always true that f is integrable on every closed bounded interval $[a, b]$.

(c) Give an example of a function $f : [a, b] \longrightarrow \mathbb{R}$ such that f is not integrable but $|f|$ is integrable.

(d) Give a definition of the natural log using an integral.

(e) Precisely state the second fundamental theorem of calculus.

(f) Derive the formula for integration by parts using the product rule.

(g) Precisely define what it means for an infinite series $\sum_{k=0}^{\infty}$ to be convergent.

(h) Is the following infinite series convergent? $\sum_{k=1}^{\infty} \frac{2^{k+2}}{3^k}$.

4. Suppose that $f : [a, b] \longrightarrow \mathbb{R}$ is continuous. Show that there is a point $c \in [a, b]$ such that

$$f(c) = \frac{1}{b-a} \int_a^b f(x)dx.$$

Hint: Use the Mean Value Theorem applied to the function $F(x) = \int_a^x f(t)dt$.

5. Suppose that $f : [a, b] \longrightarrow \mathbb{R}$ is monotone non-increasing. Prove that f is integrable without citing Theorem 5.2.1 in the text.

6. Prove directly from the definition that the Riemann integral of $f(x) = 2x + 1$ exists on the interval $[0, 1]$.

7. Prove carefully that if $f : (a, b) \longrightarrow \mathbb{R}$ is a differentiable function with bounded derivative, then f is uniformly continuous.

8. Prove that the function $f(x) = \begin{cases} x^2 & x \geq 0 \\ x\sin(x) & x \leq 0 \end{cases}$ is differentiable everywhere. You may use that $\sin(x)$ is differentiable with derivative $\cos(x)$.

9. Use the definition of the limit to prove that

$$3 = \lim_{x \longrightarrow 2} x^2 - 1.$$

10. Compute the derivative (with respect to x) of the function

$$g(x) = \int_{\sin(x)+2}^{e^x} \frac{1}{t} dt.$$

Hint: Break up the integral into two different integrals.

11. Suppose that $f : [a, b] \longrightarrow \mathbb{R}$ any function and $f(x) \geq m$ for all $x \in [a, b]$. Prove directly that

$$\underline{\int_a^b} f(x)d(x) \geq m(b - a).$$