WORKSHEET #5 – MATH 311W

OCTOBER 24TH, 2012
DUE MONDAY, OCTOBER 29TH

We will study relations. First we recall some terminology. Fix a relation $\sim$ on a set X. The elements x, y, z are elements of X.

(a) $\sim$ is called *reflexive* if $x \sim x$ for all $x \in X$.
(b) $\sim$ is called *symmetric* if whenever $x \sim y$ then $y \sim x$.
(c) $\sim$ is called *transitive* if whenever $x \sim y$ and $y \sim z$ then $x \sim z$.
(d) $\sim$ is called *antisymmetric* if whenever $x \sim y$ then $y \not\sim x$.
(e) $\sim$ is called *weakly antisymmetric* if whenever $x \sim y$ and $y \sim x$ then $x = y$. (Many references call this "*antisymmetric*")
(f) $\sim$ is called *total* if for any $x, y \in X$ we have either $x \sim y$ or $y \sim x$ (or both).

A relation is called a *partial order* if it satisfies (a) reflexive, (c) transitive and (e) weakly antisymmetric. A relation is called a *strict partial order* if it is (d) antisymmetric and (c) transitive. Finally, a relation is called a *total order* if it is (c) transitive, (e) weakly antisymmetric, and (f) total.

1. Consider the empty relation $\sim = \emptyset$ on X. Which of the properties (a)-(e) are satisfied. Is it a partial order or a strict partial order or neither?

2. Same question as **1.** but now assume that $\sim = X \times X$.

3. Prove that every antisymmetric relation is weakly antisymmetric.

4. Consider $X = \mathbb{Z}$. Write $x \sim y$ if $x|y$. What properties (a) – (f) does this satisfy? Is it a partial, strict partial, or total order?

5. Let $X = \mathbb{R} \times \mathbb{R}$. Define a relation on X by saying that $(a, b) \sim (c, d)$ if $a \geq c$ and $b \geq d$. Prove that $\sim$ is a partial order but not a total order.