This is very much like what the first page of the exam will be.

1. Short answer questions.

(a) Give an example of a surjective function but non-injective function \( f : \mathbb{Z} \rightarrow \mathbb{Z} \).

(b) What does it mean for a relation on a set to be reflexive?

(c) Is the proposition “If \( 1 + 2 = 2 \) then \( 3 + 3 = 7 \)” true?

(d) What is the power set of \( \{\emptyset\} \)?

(e) Is \( \neg(q \lor q) \leftrightarrow \neg q \land \neg p \) a tautology?

(f) Consider the function \( g : \{1, 2\} \rightarrow \mathbb{R} \) which is defined by \( g(x) = x^3 \). Is \( g \) surjective? Is \( g \) injective?

(g) If \( S \) is a set, is it always true that \( \emptyset \subseteq S \)?
2. Short answer questions continued.
(a) Give an example of an uncountable set.

(b) Is \( \mathbb{Z} \times \{1, 2, 3\} \) countable?

(c) Consider the proposition \( \exists x \forall y (Q(x, y) \rightarrow P(x, y)) \). Express the negation of the proposition in such a way that there is no negation sign outside of a quantifier, or outside of parentheses.

(d) Is it always true that \( S \neq \mathcal{P}(S) \)?

(e) How many injective functions are there from \( A = \{1\} \) to \( B = \{2, 3, 4\} \)?

(f) Do the even integers have the same cardinality as the rational numbers?

(g) Give an example of a relation on the set \( A = \{1, 2, 3\} \) that is not transitive.

(h) Is the assertion \( \{1, 2\} \in \{\emptyset, 1, 2\} \) true?
Here’s a couple problems on sets that are similar to what you might see on the exam.

3. Suppose that $A, B$ and $C$ are sets. Prove carefully (using complete sentences) that

$$(A \cap B) \cup (A \cup B^c)^c = B$$

Recall that $S^c$ denotes the complement of a set $S$.

4. Suppose that $A, B$ and $C$ are sets and that $A \neq \emptyset$. Prove that $B = C$ if and only if $A \times B = A \times C$. 