WORKSHEET #1 – MATH 2200
SPRING 2018

DUE FEBRUARY 7TH, 2018

You may work in groups of up to 4. Only one worksheet needs to be turned in per group, but make sure everyone's name is on the worksheet.

Recall the following definitions.

Definition. A function $f : A \longrightarrow B$ is called *injective* (or one-to-one) if $f(a) = f(a')$ implies $a = a'$. A function $f : A \longrightarrow B$ is called *surjective* (or onto) if for every $b \in B$, there exists an $a \in A$ such that $f(a) = b$. A function is called *bijective* if it is both injective and surjective.

1. Show that if $f : A \longrightarrow B$ and $g : B \longrightarrow C$ are functions such that $g \circ f$ is injective, then f is injective.

2. Give an example of functions $f : A \longrightarrow B$ and $g : B \longrightarrow C$ such that $g \circ f$ is injective but g is not injective.

3. Suppose that $f : A \longrightarrow B$ and $g : B \longrightarrow C$ are functions and that $g \circ f$ is surjective. Is it true that f must be surjective? Is it true that g must be surjective? Justify your answers with either a counterexample or a proof.

4. Suppose that $A = \{\star, 2, \emptyset\}$, $B = \{\{\emptyset\}, \{1, 2\}\}$. Let f be the function defined by $f(\star) = \{\emptyset\}$, $f(2) = \{\emptyset\}$ and $f(\emptyset) = \{1, 2\}$. What is the graph of f?

5. Give an example of the following functions:

(a) A function $f : \mathbb{Z}_{>0} \longrightarrow \mathbb{Z}$ that is injective but not surjective.
(b) A function $g : \mathbb{Z}_{>0} \longrightarrow \mathbb{Z}$ that is surjective but not injective.
(c) A function $h : \mathbb{Z}_{>0} \longrightarrow \mathbb{Z}$ that is bijective.