QUIZ #3 – MATH 2200
SPRING 2018

FEBRUARY 23RD, 2018

1. Let $S = \mathbb{R}_{\geq 0}$ and let $T = \{n^2 \mid n \in \mathbb{Z}_{>0}\}$.

(a) Is S finite, countably infinite or uncountable?
(b) Is T finite, countably infinite or uncountable?
(c) Is $S - T$ finite, countably infinite or uncountable?
(d) Is $T - S$ finite, countably infinite or uncountable?

You don't need to justify your answer, but it might be helpful to recall the following facts that we proved in the worksheet on countability.

(i) If A and B are countably infinite sets, then so is $A \cup B$.
(ii) If A and B are countably infinite sets, then so is $A \times B$.
(iii) If A is uncountable and $B \subseteq A$ is countably infinite, then $A - B$ is uncountable.
(iv) If A and B are uncountable, it is possible that $A - B$ is finite, or countably infinite, or uncountable (it just depends on what the sets are).
(v) For any set S, $|S| \neq |\mathcal{P}(S)|$.

Solution:

(a) uncountable
(b) countably infinite
(c) uncountable
(d) finite (in fact empty)

2. Let $S = \mathbb{R}$. We define a relation on $\mathbb{Q}$ as follows. $a \sim b$ if and only if $a + b = 0$. Is $\sim$ symmetric, transitive, or reflexive? Make sure to justify your answer with a proof (if it is) or with a counter-example (if not).

Solution: **symmetric** It is symmetric. Indeed, suppose that $a \sim b$, then $a + b = 0$, so that $b + a = 0$ and thus $b \sim a$ as claimed.
transitive It is not transitive. Note that $1 \sim -1$ and $-1 \sim 1$ but 1 is not related to 1.
reflexive It is not reflexive. Note that 1 is not related to 1.

3. Suppose that $f : A \longrightarrow B$ and $g : B \longrightarrow C$ are surjective functions. Prove carefully that $g \circ f$ is also surjective.

Solution: We want to show that $g \circ f : A \longrightarrow C$ is surjective. Thus fix $c \in C$, we want to find $a \in A$ so that $(g \circ f)(a) = c$. First since g is surjective, there exists some $b \in B$ so that $g(b) = c$. Next, since f is surjective, there is some $a \in A$ so that $f(a) = b$. Now consider $(g \circ f)(a)$:

$$(g \circ f)(a) = g(f(a)) = g(b) = c$$

by using what we have already written. Thus we have found our desired $a \in A$ which completes the proof.