WORKSHEET #5 – MATH 1260
FALL 2014

NOT DUE, OCTOBER 7TH

1. First we begin with short answer questions.

(a) Are the vectors $\langle 1, 2, 3\rangle$ and $\langle -3, -2, -1\rangle$ perpendicular?

(b) Find a vector that is perpendicular to $\langle 1, 2, 3\rangle$.

(c) True or false, the projection of a vector onto the xy-plane is always a unit vector.

(d) Find the area of the parallelogram defined by the vectors $\langle 1, 2\rangle$ and $\langle -1, 3\rangle$.

(e) Find a vector $\vec{w}$ so that if $\vec{u} = \langle 1, 0, -1\rangle$ and $\vec{v} = \langle 0, 0, 2\rangle$, then $\{\vec{u}, \vec{v}, \vec{w}\}$ form a *linearly dependent set.*

(f) Find a vector $\vec{w}$ so that if $\vec{u} = \langle 1, 0, -1\rangle$ and $\vec{v} = \langle 0, 0, 2\rangle$, then $\{\vec{u}, \vec{v}, \vec{w}\}$ form a *spanning set.*

(e) Setup, but do not evaluate, an integral which computes the arclength of $t \mapsto \langle \cos(t), t\sin(t), t^2\rangle$ for t from 2 to 3.

(f) If an ant is climbing down a hill whose height is given by $z = x^2 + y^2 + 3x\cos(y^2)$ and is at position $(1, 0)$, what direction should the ant climb to descend the hill fastest?

(g) Find the curvature of the space curve $t \mapsto \langle t, t^2, t^3\rangle$ at the point $\langle 2, 4, 8\rangle$.

We continue **1.**

(h) True or false, the normal vector is never a unit vector.

(i) Consider the following integral

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (x^2 + y^2 + 6)dydx$$

Set it up in polar coordinates (but do not evaluate it).

(j) Compute the cross product $\langle 0, -1, 2\rangle \times \langle 1, 0, 3\rangle$.

(k) Find the equation of the tangent plane to the surface $z = x^2 + y^2$ at the point $(1, 1, 2)$.

(l) Suppose $t \mapsto \vec{r}(t)$ is a parameterization of a space curve. True or false $\vec{r}'(t) \cdot \vec{N}(t) = 0$.

(m) Give an example of a surface $z = f(x, y)$ where

$$\lim_{(x,y) \longrightarrow (0,0)} f(x, y)$$

does not exist but

$$\lim_{x \longrightarrow 0} f(x, 0) \text{ and } \lim_{y \longrightarrow 0} f(0, y)$$

do exist.

(n) Suppose that we are given a function $f(x, y)$ with $\nabla f(1, 1) = \langle -1, 2\rangle$ describing the height of a hill. Further suppose that the xy-coordinates of a person is given by $t \mapsto \langle g(t), h(t)\rangle$. If $g(3) = \langle 1, 1\rangle$ and $g'(3) = \langle 0, 1\rangle$, is the person ascending or descending the hill at time $t = 3$?

(o) State the second derivative test for finding the maxes or mins of $z = f(x, y)$.

(p) If $\nabla f = \langle 3, 2\rangle$, what is the directional derivative of f in the direction $\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\rangle$?

2. The base of an aquarium of volume V is made of stone and the sides are glass. If stone costs 5 times as much as glass, what dimensions should the aquarium be (in terms of V) in order to minimze the cost of materials. Justify your answer.

3. Find the local maximums and local minimums of the following surface

$$z = xy + \frac{1}{x} + \frac{1}{y}$$

4. Find the distance of the point $(1, 2, 3)$ from the tangent plane to the surface $z = x^3 + y^3 + xy$ at $(1, 1, 3)$.

5. Reparameterize the space curve $t \mapsto \langle 2t + 1, 3t, -t \rangle$ with respect to arc length.

6. Find the volume of the solid bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 2y, z = 0, y = 4$.

7. Sketch the region of integration of the following integral

$$\int_1^2 \int_0^{\ln x} x dy dx$$

and then rewrite the integral as

$$\int_a^b \int_{g_1(y)}^{g_2(y)} x dx dy.$$

In particular, find the constants a, b and the functions $g_1(y)$ and $g_2(y)$.

8. Setup an integral to find the volume of the solid enclosed by the parabolic cylinders $y = 1 - x^2$, $y = x^2 - 1$ and the planes $x + y + z = 2$ and $2x + 2y - z + 10 = 0$.