WORKSHEET #3 – MATH 1260
FALL 2014

DUE WEDNESDAY, SEPTEMBER 17TH

For this assignment, you are allowed and encouraged to work in groups. Each group only has to turn in one assignment worksheet, but make sure it is done neatly.

Suppose we have a space curve parameterized by $t \mapsto \vec{r}(t) = \langle f(t), g(t), h(t) \rangle$. We saw in class on Wednesday that if we set $\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$ then the derivative of $\vec{T}(t)$ was perpendicular to $\vec{T}(t)$. We set $\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$, this is called the *normal vector*.

1. Verify that again by writing down the argument here.

The question that was brought up was, well then what direction is $\vec{T}'(t)$ pointing in?!? The goal of this worksheet is to provide an answer to that question.

2. Let's begin with an example. Consider the function $\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$. Compute $\vec{N}(t)$, including at some explicit examples, and then draw a picture showing the direction it is pointing. Do the same for the function $\vec{u}(t) = \langle t\cos(t), t\sin(t), t \rangle$ if you are feeling ambitious.

3. Suppose that $\vec{r}(t)$ is the equation of a line. What is $\vec{T}'(t)$, does $\vec{N}(t)$ even make sense?
Hint: You can try an example like $\vec{r}(t) = \langle 1, 2, 3 \rangle t + \langle 4, 5, 6 \rangle$ if the answer isn't clear to you.

4. Imagine that $\vec{r}(t)$ is the position of a particle at time t. Then $\vec{r}'(t) = \vec{v}(t)$ is the velocity vector of the particle and $\vec{r}''(t) = \vec{a}(t)$ is the acceleration vector. Show that $\vec{a}(t)$ is in the span of $\vec{T}(t)$ and $\vec{N}(t)$.

Hint: Let $v(t) = |\vec{v}(t)|$ be the speed function. Consider then the formula $v(t)\vec{T}(t) = \vec{v}(t)$ and differentiate it.

5. Explain in words why this must mean that $\vec{T}'(t)$ is pointing along the radial line of the acceleration.

6. Consider now the formula for curvature $\kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}$. Use this and the work you did in **4.** to derive the formula

$$\vec{a}(t) = v'(t)\vec{T}(t) + \kappa(t)(v(t))^2\vec{N}(t).$$