WORKSHEET #2 – MATH 1260
FALL 2014

DUE MONDAY, SEPTEMBER 8TH

For this assignment, you are allowed and encouraged to work in groups. Each group only has to turn in one assignment worksheet, but make sure it is done neatly.

1. Fix a nonzero vector $\vec{u} = \langle a, b, c \rangle \in \mathbb{R}^e$. Consider all of the vectors with tail at the origin and perpendicular to $\vec{u}$. What do the end points of these vectors look like? Write down an equation in x, y, z to describe this set.

Generally a function $ax + by + cz = 0$ defines a plane through the origin and perpendicular to this plane is the *normal vector* $\langle a, b, c \rangle$.

2. Find the equation of a plane perpendicular to $\vec{u} = \langle a, b, c \rangle$ and passing through the point (d, e, f).

Hint: Try shifting the equation we considered above.

3. Given a direction vector $\vec{v} = \langle x, y, z \rangle$ and starting point $Q = \langle a, b, c \rangle$ consider

$$t\vec{v} + Q = t\langle x, y, z \rangle + \langle a, b, c \rangle = \langle xt + a, yt + b, zt + c \rangle.$$

As the parameter t varies, what points does this expression hit? Describe the answer geometrically.

Hint: If you get stuck, try making up some numbers x, y, z, a, b, c and plugging in $t = 0, 1, 2, \ldots$.

4. What you did in problem **3.** was to write down a *parametric equation of a line.* Consider the line L parameterized by $t \mapsto t\langle 2, 4, 6\rangle + \langle 0, 0, 1\rangle$. Find two other parameterizations of the *same* line: one where $t = 0$ gives the same point as the parametrization above and the other where $t = 0$ gives a different point.

5. Consider two planes passing through the origin: $ax + by + cz = 0$ and $dx + ey + fz = 0$. If the planes are different, then they intersect in a line (convince your group of this). Find a parameterization of this line.

Hint: The line will be orthogonal to the normal vectors of both planes, have we learned any way recently to cook up a vector orthogonal to two other vectors?

6. Now consider the following two planes $2x + y - z = 1$ and $x - 3y + 2z = 1$. Are the planes parallel, why or why not? If not, find a parameterization of the line that passes through both of them.

Hint: You can use the same strategy as above to find the direction of the line of their intersection. Then you just need to find a common point on the line.

7. Suppose you are given a plane H defined by $ax + by + cz = 0$ and a vector $\vec{u} = \langle x, y, z\rangle$. Write here what the phrase *the projection of $\vec{u}$ onto H* should mean (draw a pretty picture). Once you have figured that out, write down a general formula for the projection of $\vec{u}$ onto H.

Hint: For the second part, it's easy to project $\vec{u}$ onto the normal vector of the plane. Once you have done that, do some basic vector arithmetic to find the projection of $\vec{u}$ onto H.

8. Where does the parameterized line $t \mapsto t\langle -1, 2, -3\rangle + \langle 0, 1, 0\rangle$ intersect the plane $2x+3y-4z = 0$?

9. Consider the two parameterized lines $s \mapsto s\langle 1, 2, 3\rangle$ and $t \mapsto t\langle 1, 0, -1\rangle + \langle 0, 4, 8\rangle$. Do they intersect, and if so, where?

10. Find the distance of a point $Q = \langle 0, -1, 3\rangle$ away from the plane $2x + y - z = 2$.

Hint: Parameterize a line starting from Q and perpendicular to the plane. Find where that line intersects the plane.