WORKSHEET #10 – MATH 1260
FALL 2014

NOT DUE, NOVEMBER 21ST

1. First we begin with short answer questions.

(a) Define the term *open cover*.

(b) Define the term *compact*.

(c) Write down an open cover of the open interval $(0, 1)$ that does not have a finite subcover.

(d) Write down a precise definition of what it means for a function $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ to be continuous.

(e) If $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$, for $t = 0$ to 3, is a parameterization of a curve in $\mathbb{R}^3$, write down an integral that would compute its arclength.

(f) State Green's theorem.

(g) State the divergence theorem.

We continue **1.**

(h) Is the following vector field the gradient of a potential function? $\vec{F} = \langle y, y, \cos(z)\rangle$

(i) Rewrite the integral

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}} x^2 y dz dy dx$$

in spherical coordinates (do not evaluate or simplify unless you want to).

(j) Is the following vector field the curl of some other vector field $F = \langle ye^{\cos(z)}, x^2\frac{1}{1+x^2+z^2}, e^x * y\rangle$?

(k) Use Green's theorem to compute the work done by the vector field $\vec{F} = \langle y, 2x\rangle$ on a particle that moves in a circle of radius 2 around the point $\langle 23, 111111\rangle$.

(l) Give an example of a vector field that is not conservative.

(m) Setup an integral to compute the volume of a region below the $z = x + y + 10$ plane, above the parabaloid $z = -5 + x^2 + y^2$ and inside the cylinder $x^2 + y^2 = 1$. Do not evaluate the integral.

(n) Parameterize the part of the parabaloid $z = x^2 + y^2$ that lies above the triangle with vertices $\langle 0, 0, 0\rangle, \langle 0, 1, 0\rangle, \langle 1, 0, 0\rangle$. Use a function $\vec{r}(u, v)$ and make sure to specify the domain that the u, v are allowed to come from.

2. Consider a particle moving along a curve parameterized by $\vec{r}(t) = \langle t(t-1)e^{\sin(t)} + t^2, 1 - t - (e^t - 1)(t-1)\sin(e^t)\rangle$ for $t = 0$ to 1. Find the work done by the force field $\vec{F} = 2xy\vec{i} + (x^2+1)\vec{j}$ as the particle moves along this curve segment.

3. Consider the region above the plane $z = -1$, below the plane $z = 2x$ and inside the cylinder $x^2 + y^2 = 1$ (in other words, $z \geq -1, z \leq x, x^2 + y^1 \leq 1$). Draw the region. Suppose now that the density of the object is given by the formula $\rho(x, y, z) = 5\cos(x)e^z$. Setup, but do not evaluate an integral that computes the mass of the region.

4. Let S be the surface defined by the equation

$$z = x(1-x)y(1-y)(7+\sin(xy)+e^{\cos(x)})$$

and above the square $0 \leq x \leq 1$, $0 \leq y \leq 1$. Let $\vec{F}$ be the vector field $y^2\vec{i} - \vec{j} + x\vec{k}$. Compute the flux integral

$$\iint_S \vec{F} \cdot dS.$$

5. Suppose an extremely complicated surface S (with upward orientation) defined by a (continuously differentiable) equation $z = f(x, y)$ is always ≥ 2 and intersects the $z = 2$ plane in a circle of radius 2 centered at the point $\langle 0, 0, 2 \rangle$. Further suppose that the volume of the region above the plane $z = 2$, below the surface $z = f(x, y)$ and above the aforementioned circle, is equal to 3. Compute

$$\iint_S \langle 3x, e^z, y+1 \rangle \cdot dS.$$