COMPUTER #3 (RETURN TO THE DOUGHNUT) – MATH 1260, FALL 2014

DUE FRIDAY OCTOBER 24TH

Grading and rubric

Each group should turn in a 2-5 page written report explaining what was done mathematically. It needs to include the following:

(1) The introduction (and introduction to each section) needs to clearly restate the problem to be solved.
(2) Citations and/or restatements of any significant or unusual formulas.
(3) Use graphs and tables where appropriate, or short snippets of Mathematica/Maple/Maxima code.
(4) Include proper spelling and grammar and use of mathematical formulas.
(5) The mathematics needs to be complete and correct.

If that is done, you can receive a 17/20. To receive the 3 final points, you need to do something above and beyond the assignment, in a *mathematical* way. Simply copying a couple paragraphs from the internet will not suffice.

Return to the doughnut

After locating the sprinkles and narrowly avoiding being killed with insecticide, the pair of ants return to the queen and report on the doughnut. After telling the queen that the units of the doughnut are in meters, the queen decides to relocate the colony to within the doughnut itself. Before this can happen though, several pieces of mathematics need to be worked out first.

1. The ants need a coordinate system to live within the doughnut. It should function as follows. The first coordinate α will denote the angle along the main radius 4 circle ($x^2 + y^2 = 4^2$). The second coordinate β will denote the angle along the radius 2 circle (the one you would see if you cut into the doughnut). The third and final coordinate r should denote the distance you are away from the main radius 4 circle (ie, the distance you traveled along the angle β).

Write down functions $x = a(\alpha, \beta, r), y = b(\alpha, \beta, r), z = c(\alpha, \beta, r)$.

Find the volume of the doughnut using this coordinate system (making sure to compute the jacobian term). Compare this to the volume you get via the revolution technique from single variable calculus.

2. Using the parameterization of the surface of the doughnut from the previous assignment, compute the surface area of the doughnut. The surface area of a surface parameterized by $\langle s, t\rangle \mapsto \langle f(s,t), g(s,t), h(s,t)\rangle$ is given by the formula:

$$\int\int_D |\langle f_s(s,t), g_s(s,t), h_s(s,t)\rangle \times \langle f_t(s,t), g_t(s,t), h_t(s,t)\rangle| dsdt.$$

Take a paragraph to explain where this integral comes from (remember what the length of a cross product measures).

3. The interior of the doughnut contains jelly, unfortunately this jelly is not uniformly distributed. In particular the radius of the jelly (around the central circle) is 1 meter at the α-angles of

$0, \pi/2, \pi, 3\pi/2$ and drops to 0.2 meters at $\alpha = \pi/4, 3\pi/4, 5\pi/4, 7\pi 4$. Estimate this using an appropriate trigonometric function. Then find the volume of the jelly using your approximation.

Finally, if the jelly is three times denser than the dough, what percentage of the doughnut's weight is accounted for by the jelly.

4. The critical mass of sprinkles at $Q = (3\sqrt{2}, 3\sqrt{2}, 0)$ appears to have negative effects on young ants so the plan is to locate the colony at the opposite edge at

$$P = (-3\sqrt{2}, -3\sqrt{2}, 0).$$

However, it is still necessary to travel back and forth from the colony and the sprinkles frequently. The ants can burrow through the doughnut, although this takes 50% longer, but may not burrow through the jelly. Compare at least three different paths between these locations, including some paths that spend some of their time burrowing inside the doughnut. Make sure to include computer generated graphs of these paths (if you need help on this, ask).

Ideas for going above and beyond

You don't have to do any of these, or you can come up with your own. But here are some ideas that lead to 20/20 points.

(1) You could try to figure out how much this doughnut would actually weigh and analyze other information about it. For instance, what would it cost to actually make?

(2) Can you figure out the number of calories in this doughnut (make some reasonable estimates, look up how many calories per unit volume). How long could an ant colony of size n ants survive in the doughnut (again, you'll need to make assumptions about the type of ants, you'll need to figure out how much they eat, etc).

(3) You could take a serious stab at trying to find optimal paths. You could analyze a larger number of possible paths including some more interesting ones (that do not look like line segments on various parameterizations).

(4) You could investigate a more interesting doughnut shape (for instance, can you figure out how to parameterize a doughnut with multiple holes, or a doughnut that has an interesting knot in it?

(5) Whatever else you can think of.