Name ______________________

Math 217 Winter, 2007

Midterm 2

Problem	Possible Points	Actual Points
1	10	
2	10	
3	10	
TRUE/FALSE	20	
Total	50	

In all the problems (with the exception of the TRUE/FALSE), indicate how you arrived at your answer. The answer alone will get at best partial credit.

Problem 1 (10 points)

a) Compute the determinant of $A = \begin{bmatrix} 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \\ -5 & -8 & 0 & 3 \\ 0 & 5 & 0 & 6 \end{bmatrix}$.

(**Hint**: although you should still justify your answers for parts b–e, you shouldn't have to do any further calculations.)

b) Is A invertible?

c) What is the dimension of $\mathrm{Nul}(A)$?

d) What is the rank of A?

e) Are the columns of A linearly independent or dependent?

Problem 2 (10 points) Let $A = \begin{bmatrix} 1 & -3 & 0 & 2 & 4 \\ -2 & 6 & -3 & -1 & -11 \\ 1 & -3 & 2 & 0 & 6 \end{bmatrix}$.

a) Find bases for Nul(A), Col(A), and Row(A).

b) Suppose $V = \text{Span}\{p_1(t), p_2(t), p_3(t)\}$, where

$$\begin{aligned} p_1(t) &= 1 - 3t + 2t^3 + 4t^4, \\ p_2(t) &= -2 + 6t - 3t^2 - t^3 - 11t^4, \\ p_3(t) &= 1 - 3t + 2t^2 + 6t^4. \end{aligned}$$

Is V a subspace of $\mathbb{P}_4$? If so, what is its dimension?

Problem 3 (10 points) Suppose $\mathcal{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ and $\mathcal{C} = \{\mathbf{c}_1, \ldots, \mathbf{c}_n\}$ are bases for a vector space V. Show that $\{[\mathbf{b}_1]_\mathcal{C}, \ldots, [\mathbf{b}_n]_\mathcal{C}\}$ is a basis for $\mathbb{R}^n$.
(**Hint 1**: You are welcome to apply any of the theorems about bases of finite-dimensional vector spaces here.)
(**Hint 2**: (unrelated to hint 1) Recall that $[\mathbf{b}_1]_\mathcal{C}, \ldots, [\mathbf{b}_n]_\mathcal{C}$ are the columns of the change-of-basis matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$.)

TRUE/FALSE (20 points)

Below are ten assertions. For each, circle either T or F to indicate whether you believe the assertion is always TRUE or sometimes FALSE. There is no need to justify your response.

T F A vector space spanned by an infinite set is infinite-dimensional.

T F Let M_n be the vector space of all $n \times n$ matrices (you may want to convince yourself this is a vector space). The transformation $T : M_n \to \mathbb{R}$ defined by $T(A) = \det(A)$ is a linear transformation.

T F The parallelogram defined by vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ has area $\det([\mathbf{v}_1 \quad \mathbf{v}_2])$.

T F In the L-U decomposition of a matrix A, the matrix L is invertible.

T F Let V be the vector space of differentiable functions $f : \mathbb{R} \to \mathbb{R}$. We showed that differentiation is a linear transformation, $D : V \to V$, $D(f) = f'$. Then D^{-1} is the linear transformation given by integration.

T F The set of all vectors of the form $\begin{bmatrix} a - 3b \\ 2(a+b) \\ b - a \\ 4b \end{bmatrix}$, where a, b in $\mathbb{R}$, is a plane in $\mathbb{R}^4$.

T F Let $\mathcal{B} = \{\mathbf{b}_1 = 1, \mathbf{b}_2 = t, \mathbf{b}_3 = t^2\}$, and let $\mathcal{C} = \{\mathbf{c}_1 = 1, \mathbf{c}_2 = 1 + t, \mathbf{c}_3 = 1 + t + t^2\}$, be bases for $\mathbb{P}_2$. Then $\det\left(\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}\right) \neq \det\left(\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}\right)$.

For each of the following, let A be an $m \times n$ matrix.

T F $\text{Col}(A) = \text{Row}(A^T)$.

T F $\dim(\text{Row}(A)) = n - \dim(\text{Nul}(A))$.

T F The solution set to $A\mathbf{x} = \mathbf{b}$ is a subspace of $\mathbb{R}^n$.