EXTRA CREDIT #2

MATH 217 – SECTION 4

One thing we never really proved in class was the following fact about determinants and row operations.

Desired Result If A is an $n \times n$ matrix, then if B is obtained from A by doing a single elementary row operation, then

- $r \det A = \det B$ if that row operation was scaling a single row by r,
- $\det A = -\det B$ if that row operation was row interchange,
- $\det A = \det B$ if that row operation was row replacement.

We want to prove each of these facts. We've basically seen it for the case when A is an elementary matrix, but let's do the general case.

1. First verify these facts whenever A is a 2×2 matrix. For example, to verify the first case, take an arbitrary matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

and scale the first row by a number r. Now compute the determinant of this new matrix and compare it to $\det A$. Also try scaling the second row. Check all three operations for 2×2 matrices. (2 points total)

Suppose we have an $n \times n$ matrix A where $n > 2$. Let B be a matrix obtained by doing a single row operation to A.

2. Explain why there is always at least one row of B that is the same as the corresponding row of A. (1 point)

3. Use a cofactor expansion along this identical row, to express the determinant of A and to express the determinant of B. How are the matrices A_{ij} and B_{ij} that appear in each of these computations related? (2 points)

4. Explain why the observation you made in problem #3 (when combined with problem #1) is enough to prove the desired result for 3×3 matrices. (1 point)

5. Explain how you would prove it for 4×4, 5×5, 6×6 and $n \times n$ matrices. (1 point).

In this last step, you have basically done a proof strategy called "mathematical induction".