NOTES ON INJECTIVE AND SURJECTIVE FUNCTIONS
MATH 186–1
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First we recall the definition of a function.

Definition 0.1. A *function* is the following information.

(a) A *domain* $= D$. In other words, a set of allowable input values.
(b) A *codomain* $= C$. In other words, a set of allowable output values.
(c) A set S of pairs (a, b) where $a \in D$ and $b \in C$ satisfying the following condition. If (a, b) and (a, c) are both in S, then $b = c$ (ie, the pairs are the same).

A function is typically denoted by a single letter, many times f, and is often writen $f : D \to C$ where D is the domain and C is the codomain. Furthermore, if (a, b) are in the set, then we write $f(a) = b$. Note that the sets involved need not be sets of real numbers. Functions that input and output real numbers often use the letters f, g, h.

Definition 0.2. A function $f : D \to C$ is called *injective*[1] if $f(a) = f(a')$ implies that $a = a'$. In other words, associated to each possible output value, there is AT MOST one associated input value.

Definition 0.3. A function $f : D \to C$ is called *surjective*[2] if for every $b \in C$, there exists an $a \in D$ such that $f(a) = b$. In other words, associated to each possible output value, there is AT LEAST one associated input value.

Definition 0.4. A function $f : D \to C$ is called *bijective* if it is both injective and surjective. In other words, associated to each possible output value, there is EXACTLY ONE associated input value.

Now also recall composing functions.

Definition 0.5. Suppose that $f : B \to C$ is one function and $g : A \to B$ is another function. One can form the composition $f \circ g : A \to C$ defined by the following rule:

$$f \circ g(a) = f(g(a)).$$

We know the following facts about injective and surjective functions. You should be able to prove all of these results to yourself (proofs will not be provided here).

Proposition 0.6. *Suppose that $f : B \to C$ and $g : A \to B$ are functions. Then we know the following facts:*

(1) *If $f \circ g$ is injective, then g is injective.*
(2) *If $f \circ g$ is surjective, then f is surjective.*
(3) *If f and g are injective, then $f \circ g$ is injective.*
(4) *If f and g are surjective, then $f \circ g$ is surjective.*

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[1] Injective functions are sometimes called *one-to-one*.

[2] Surjective functions are sometimes called *onto*.