HOMEWORK #8

DUE FRIDAY MARCH 26TH

(1) Are the following matrices diagonalizable? If so, find a diagonal matrix similar to them. If not, justify why they aren't diagonalizable.

(a) $\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (Hint, look at the solutions to an old worksheet)

(2) Suppose A and B are similar 2×2 matrices and C is another 2×2 matrix. Is AC similar to BC? Prove or give a counter example.

(3) Explain in words why the following sentence doesn't make sense.

Suppose $S, T : \mathbb{R}^2 \to \mathbb{R}^2$ are two similar linear transformations.

(4) An important operation which can be done to matrices is the trace. Given a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, we define the trace of A to be $\mathrm{Tr}(A) = a + d$.

(a) Prove directly that for any 2×2 matrices A and B, we have that $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$.

(b) Prove that if A and B are similar matrices, then $\mathrm{Tr}(A) = \mathrm{Tr}(B)$.

(c) Suppose that A is a matrix whose associated linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ has two linearly independent eigenvectors. Prove that $\mathrm{Tr}(A)$ is the sum of the eigenvalues of A.

(5) Let A be a matrix with an eigenvalue λ and suppose that $q(x)$ is a polynomial. Show that $q(\lambda)$ is an eigenvalue associated with the matrix $q(A)$.

(6) Write the following complex numbers in the form $a + bi$ where $i = \sqrt{-1}$.

(a) $1/(1 + i)$.

(b) $\sqrt{-8} - \sqrt{2}$.

(c) $(1 - 7i)/(\sqrt{-8} - \sqrt{2})$.

(7) Find the modulus (ie absolute value) and argument of each of the following.

(a) $1/(3 + 4i)$

(b) $|2 + 3i|$

(8) For each of the matrices below, find the (possibly complex) eigenvalues and describe the (possibly complex) eigenvectors. If the matrix is diagonalizable (with complex entries), diagonalize it. Otherwise, prove it can't be done.

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -1 \\ 13 & 5 \end{bmatrix}$

(9) Prove that if ω is an nth root of one, then so is ω^k for any integer k.

(10) A number ω is called a *primitive nth root of unity* if $\{1, \omega, \omega^2, \ldots, \omega^{n-1}\}$ is the set of all nth roots of 1. How many primitive nth roots of unity are there for $n = 3, 4, 8, 10$? Justify your answer.