HOMEWORK #5

DUE FRIDAY FEBRUARY 12TH

(1) Prove that if $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then $|\sum_{n=1}^{\infty} a_n| \leq \sum_{n=1}^{\infty} |a_n|$.

(2) For each of the following sequences of functions $\{f_n\}$, determine the pointwise limit f of $\{f_n\}$ if it exists. Also decide whether $\{f_n\}$ converges uniformly to this function.
   (a) $f_n(x) = e^{-nx^2}$ on $[-1, 1]$.
   (b) $f_n(x) = \frac{nx}{1+n+x}$ on $[0, \infty)$. *Hint:* Consider $|f(x) - f_n(x)|$ for x large.
   (c) $f_n(x) = x^n - x^{2n}$. *Hint:* Find the maximum of $|f - f_n|$ on $[0, 1]$.

(3) Find the taylor series at 0 of the following functions.
   (a) $f(x) = \ln(x - a)$, for $a \neq 0$.
   (b) $f(x) = \frac{1}{\sqrt{1-x^2}}$.

(4) Prove that the series $\sum_{n=1}^{\infty} \frac{x}{n(1+nx^2)}$ converges uniformly on $\mathbb{R}$.

(5) Prove that if $f(x) = \sum_{n=0}^{\infty} a_n x^n$ is an even function then $a_n = 0$ for n odd. Prove that if f is odd then $a_n = 0$ for n even.

(6) Find a sequence of integrable functions $\{f_n\}$ that converges to the non-integrable function that is 1 on the rationals and 0 on the irrationals.