HOMEWORK #2
MATH 186–1
WINTER 2010

DUE TUESDAY JANUARY 19TH

(1) Prove that $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$.
(2) Prove in detail the fact that $\sin'(\pi) = -1$ (which is equal to $\cos(\pi)$). We briefly discussed this in class (and the book gives a big hint if you read it).
(3) Prove that

$$\arcsin(a) + \arcsin(b) = \arcsin\left(a\sqrt{1-b^2} + b(\sqrt{1-a^2})\right)$$

(4) Find $\lim_{x\to\infty} \sin(\frac{1}{x})$.
(5) Prove that $|\sin(x) - \sin(y)| \le |x-y|$. Then prove that the $\le$ sign can be replaced by $<$ if $x \ne y$.
(6) Compute the derivatives with respect to x of the following functions.
   (a) $f(x) = e^{e^{e^{\sin(x)}}}$.
   (b) $g(x) = \left(\arcsin\left(\frac{e^x}{\cos(x)}\right)\right)^{\log_{10}(x)}$.
   (c) $h(x) = \int_1^x e^{\sin^2(t)}dt$.
(7) Find $\lim_{x\to 0^+} x^x$.
(8) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is differentiable and never zero.
   (a) Prove that $\log(|f|)' = f'/f$.
   (b) Suppose that $f' = cf$ for some constant c. Show that $f(x) = ke^{cx}$ for some number k.
(9) Prove that

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} \le e^x$$

for any $x \ge 0$ and any integer $n > 0$. Hint: Use induction on n and take derivatives.
(10) Suppose that f satisfies $f' = f$ and $f(x+y) = f(x)f(y)$ for all x and y. Prove that $f(x) = e^x$ or that $f = 0$.