HOMEWORK #7
MATH 185–4
FALL 2009

DUE TUESDAY, NOVEMBER 3RD

Only use the definition of the derivative for these problems. DO NOT use the chain rule or product rule, etc.

(1) Suppose that $f(x) = x^2$.
 (a) What is $f'(9)$?
 (b) What is $f'(3^2)$?
 (c) What is $f'(a^3)$?
 (d) What is $f'(x^7)$?
 (e) Set $g(x) = f(x^2)$. Is it true that $g'(x) = f'(x^2)$ for all x? Prove or disprove.

(2) Suppose that $f(x)$ is differentiable at a. For the following two problems, you must write out the functions carefully and correctly.
 (a) Let $h(x) = f(x/3)$. Prove that $h'(a) = f'(a/3)/3$.
 (b) Let $g(x) = f(x-1)$. Prove that $g'(a) = f'(a-1)$.

(3) Let

$$f(x) = \begin{cases} x^2, & x \text{ rational} \\ 0, & x \text{ irrational} \end{cases}$$

Prove that $f(x)$ is differentiable at the origin (ie, at $a = 0$).

(4) Suppose that f is an even function (that is, $f(x) = f(-x)$ for all x in its domain) and further suppose that f is differentiable. Prove that $f'(x)$ is odd. That is, prove that $f'(-x) = -f'(x)$. Draw a picture to help convince yourself that the statement is true. *Hint:* Let $g(x) = f(-x)$ and then find $g'(x)$ (using the definition of the derivative). But notice that $g(x) = f(x)$ by the even hypothesis so that $g'(x) = f'(x)$.

(5) Suppose that g is an odd function. Prove that g' is an even function.

(6) Prove the following formula by induction. $1^3 + \cdots + n^3 = (1 + \cdots + n)^2$.

(7) Prove the following formula by induction. $x + x^2 + \cdots + x^k = \frac{x - x^{k+1}}{1-x}$.

(8) If $S_n(x) = x^n$ and $0 \le k \le n$, prove that

$$S_n^{(k)}(x) = \frac{n!}{(n-k)!} x^{n-k}$$

Hint: Fix n and perform an induction on k.

(9) From Kaplan-Lewis 1-8, do problem #7. From 1-9, do problem #7. From 1-11, do problems #7, #8.