HOMEWORK #5
MATH 185–4
FALL 2009

DUE FRIDAY OCTOBER 9TH

(1) Suppose you are given a function g that is defined and continuous on $[0,5]$. Suppose that $g(0) = 2$, $g(1) = 10$, and $g(3) = -1$. Prove that g is *NOT* one-to-one.

(2) Show that there is a number x such that
$$x^{43} + \frac{7}{|\cos(x)| + 1} = 61234.63 - \pi^2$$

(3) Suppose that $f(x)$ is continuous on $[0,1]$ and that the image (ie range) of f is also a subset of $[0,1]$. Prove that $f(x) = x$ for some $x \in [0,1]$.

The next few problems involve the *least upper bound axiom*

(4) Find the least upper bound and the greatest lower bound (if they exist) of the following sets. Also decide which sets have greatest and least elements (ie, when the least upper bound / greatest lower bound happens to belong to the set).
  (i) $\{1/n | n \in \mathbb{N}\}$
  (ii) $\{1/n | n \in \mathbb{Z}, n \neq 0\}$
  (iii) $\{x | x^2 < 2\}$
  (iv) $\{x | x^2 + x - 1 < 0\}$
  (v) $\{x > | \sqrt{x} \text{ is an integer}\}$.

(5) Suppose that f is continuous on $[1,3]$ and that $f(x)$ is an irrational number for all $x \in [1,3]$. Prove that f is a constant function.

(6) A set of real numbers $A \subseteq \mathbb{R}$ is called *dense* if for every non-empty open interval (a,b), $A \cap (a,b) \neq \emptyset$ (ie, $A \cap (a,b)$ has some elements). For the rest of the problem, fix A to be a dense subset of $\mathbb{R}$. (For example, the rational numbers and the irrational numbers are both dense).
  (a) Suppose that f is a continuous function on $\mathbb{R}$. Also suppose that $f(x) = 0$ for every $x \in A$. Prove that $f(x) = 0$ for all $x \in \mathbb{R}$.
  (b) Suppose that f and g are continuous functions on $\mathbb{R}$. Further suppose that $f(x) = g(x)$ for every $x \in A$. Prove that $f(x) = g(x)$ for every $x \in \mathbb{R}$.
  (7) In Kaplan and Lewis, from section 1–6, do problems 2, 4, 5. From section 1–7, do problems 3, 4.