

HOMEWORK #4
MATH 185-4
FALL 2009

DUE FRIDAY OCTOBER 2ND

- (1) (a) Write down a definition of $\lim_{x \rightarrow a^+} f(x) = \infty$
(b) Prove that $\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$.
(c) Write down a definition of $\lim_{x \rightarrow \infty} f(x) = -\infty$
(d) Prove that $\lim_{x \rightarrow \infty} 1 - x^3 = -\infty$.
(2) Consider the function defined on $\mathbb{R}$ by the following rule:

$$f(x) = \begin{cases} x - 1, & x \geq 1 \\ 3x - 3, & x < 1 \end{cases}$$

Use ϵ 's and δ 's to prove that

$$\lim_{x \rightarrow 1} f(x) = 0.$$

- (3) Suppose that f is a function with domain $\mathbb{R}$ that satisfies $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. Further suppose that f is continuous at 0. Prove that f is continuous at every $a \in \mathbb{R}$.
(4) Give an example of a function f such that f is not continuous at *any* point, but that $|f|$ is continuous at *every point*.
(5) Suppose that f is a function that is continuous at a and that $f(a) > 0$. Show that there exists an open interval (b, c) containing a such that $f(x) > 0$ for all $x \in (b, c)$.
(6) Suppose that f is a function satisfying $|f(x)| \leq |x|$ for all x . Show that f is continuous at 0.
(7) On pages 54-56 of Kaplan and Lewis, do problems 1, 4, 9(a,b,c,g)