JUNE 14TH MATH PROBLEM SET

There are only 10 types of people in the world: those who understand binary, and those who don't.–Anonymous

The number 376 is thought of as 3 hundreds, 7 tens and 6 ones, or in other words, $3 \cdot 10^2 + 7 \cdot 10 + 6$. But we don't have to use 10. We can use another number, such as 2. For example, the number eleven can be written as $1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2 + 1$. In other words, we write eleven as 1011 in binary. But we can do other bases as well. For example, base 7, the number eleven is written as $1 \cdot 7 + 4$, or 14.

In the following questions, x_n indicates that the number x is written in base n. For example, the number twelve could be written 1100_2, 15_7, or 12_{10}. If no base is specified, the base is 10.

1. Write the base 10 equivalent to the number in the given base.

(a) 34_{12} (b) 15_6 (c) 27_{26}
(d) 88_9 (e) 1234_5 (f) 1001101_2

2. Write the given number in the given base.

(a) 100 (4) (b) 425 (5) (c) 81 (9)
(d) 15 (2) (e) 15 (7) (f) 15 (16)

3. Find a word that is prime when it is interpreted as a number written in base 26. (Here, word means string of letters. It doesn't have to be an English word, although that would be fun. Our convention has been A=0, B=1, etc.) Is your name prime? (If your name is long, you might want to try a nickname.)

4. Are there any prime words that end in the letter A? B? C? N? Z?

5. Construct the single-digit multiplication tables for bases 4, 5, and 8. Using the letters A and B for 10 and 11, construct the single-digit multiplication table for base 12. Do you see any patterns? Can you explain them?

In base 10, there are easy ways to check the divisibility of a number by 2, 3, 4, 5, 6, 9, or 11 based on its digits. Figure these out and write them down. Make sure all your group knows how to do all of them.

6. Find all the examples of how similar tests work that you can in bases 4, 5, 6, 9, 11, 12, 15, 16, 26, 30.

7. We will play a game where we see which team can do these tests the fastest.