SYLLABUS FOR MATH 5310/20

The text for this course sequence is Michael Artin's *Algebra*, Second Edition. We switched to this book in 2011, from Hernstein's insular book. (Algebra is from Mars ...) Artin's book includes topics from Lie groups, Number Theory and Algebraic Geometry. It is a great book. A motivating example for 5310 is the problem of counting rotations of a platonic solid. The group of rotations is too large to grasp at once, but the group acts transitively on the set of faces, and the stabilizer of a face is easy to figure out. Take, for example, the dodecahedron. There are 12 faces and 5 rotations stabilizing each face. Thus the order of the group is $12 \times 5 = 60$. This example relates to several topics: group actions, stabilizers, cosets, and equivalence relations.

I started the class with Chapter 2, which includes basic results in abstract groups, culminating with isomorphism theorems. Chapters 3 and 4 give an abstract treatment of Linear Algebra. Since group quotient is introduced in Chapter 2, it is easy to introduce a quotient of vector spaces and to compute its dimension. This is not done in the book, a weak point, if there is one. This, combined with the first isomorphism theorem, gives a natural proof of the notorious result in linear algebra: for an $n \times m$ matrix A, the rank plus nullity of A equals n. Chapter 4 includes an unilluminating proof of Jordan decomposition. This is best left for 5320. The main result in 5320 is a classification of finitely generated modules over a euclidean ring.

The following is the list of topics covered during the year 2012/13. In the previous year, a week was spent on the ruler and compass construction. Instead, I stated the main results of Galois theory, and worked out examples given by cyclotomic and Kummer extensions. It is also worthwhile to spend time on the class group for quadratic fields, as this is done rather nicely in the book. (GS, April 25 2013)

0.1. **Office hours.** MWF 10:30-12:00 in LCB 205.

0.2. **5310 Topics.**

(1) Groups, subgroups, homomorphisms.
(2) Equivalence relations, cosets and conjugacy classes, theorem of Lagrange.
(3) Modular arithmetic, fields and finite fields.
(4) Normal subgroups, quotient group, and the first isomorphism theorem.
(5) Vector spaces over a general field.
(6) The meaning of row column operations, dimension of vector spaces.
(7) Quotient spaces, their dimension (not in the book).
(8) Linear transformations, a brief discussion of Jordan normal form.
(9) Orthogonal transformations. Theorem of Euler for dimension 3.
(10) Group actions, application to classification of finite subgroups in $SO(3)$.
(11) The class equation and simplicity of icosahedral group.
(12) The first Sylow theorem.

0.3. **5320 Topics.**

(1) Rings and ideals. Quotient rings. Pruduct rings.
(2) Field of fractions. Maximal ideals. Nullstellensatz.
(3) Factorization. Euclidean domains.
(4) Gauss's lemma and Eisenstein's criterion.
(5) Modules and $\mathbb{Z}$-modules. Row column reduction over rings.
(6) Noetherian rings.
(7) Classification of finite commutative groups.
(8) Vector space with a linear transformation: a module. Jordan decomposition.
(9) Fields. Algebraic numbers.
(10) Class group of quadratic rings.
(11) Cyclotomic and Kummer extensions, automorphisms.

0.4. **Exercises.**

- Chapter 2: 1.1-3, 2.3, 2.4, 3.1, 3.2, 4.1-5, 4.8, 5.1, 5.2, 5.4, 5.6, 6.1, 6.2, 6.4-6, 6.8, 7.1, 7.2, 7.5, 8.1, 8.6, 8.7, 8.9, 8.11, 9.1-3, 10.3, 10.4, 11.1, 11.4, 11.9, 12.2, 12.4, 12.5.
- Chapter 3: 1.1, 1.2, 1.4, 1.5, 1.8, 1.11, 2.1, 2.2, 3.1, 3.2, 3.7, 4.1, 4.3, 4.4, 5.1.
- Chapter 4: 1.2, 1.3, 1.5, 2.1, 2.3, 3.1, 3.3, 4.1, 4.2, 4.3, 4.5, 4,6, 5.2, 5.3, 5.6, 6.2, 6.3, 6.4, 6.7, 6.8, 7.2, 7.3.
- Chapter 5: 1.1, 1.2, 1.3, 1.4.
- Chapter 6: 4.1, 4.2 (a), 7.1, 7.7, 7.9, 8.1, 8.4, 9.1, 9.2, 11.1, 11.3, 11.6, 12.3.
- Chapter 7: 2.1, 2.2, 2.7, 2.8, 2.14, 3.2, 3.3, 4.4, 4.5, 4.6, 6.2, 7.3, 7.5, 7.8.
- Chapter 11: 1.1, 1.3, 1.8, 2.1, 3.2, 3.3, 3.8, 3.9, 3.12, 4.2, 4.3, 4.4, 5.1, 5.4, 5.5, 6.1, 6.2, 6.4, 6.5, 6.8, 7.1, 7.2, 7.3, 8.2, 8.3.
- Chapter 12: 1.2, 1.4, 2.1, 2.5, 2.6, 2.8, 3., 3.2, 4.1, 4.3, 4.7, 4.8, 4.9, 5.1, 5.3, 5.5, 5.7.
- Chapter 14: 1.1, 1.4, 2.1, 2.4, 4.1, 4.3, 4.6, 7.1, 7.6, 8.2, 8.3, 4.8, 5.2, 6.2, 7.5, 7.9.
- Chapter 15: 2.1, 2.2, 3.1, 3.2, 3.3, 3.4, 3.8, 4.1, 4.2, 5.1, 5.3, 7.2, 7.3.

0.5. **Grade.** Will be based on 5 in-class exams, given every 3rd Wednesday, not counting the Fall break week: 09/07, 09/28, 10/26, 11/16, and 12/07. I will drop the lowest score exam. It is essential to do HW problems in order to do well on the exams. Approximate grade scale: A (93-100), A- (90-92), B+ (87-89), B (83-86), B- (80-82), C+ (77-79), C (73-76), C- (70-72), D+ (67-69), D (63-66), D- (60-62), E (0-59).