

2009 REU, JUNE 08 - JUNE 26

GORDAN SAVIN, OFFICE LCB 205 AND AARON WOOD

Overview: We plan to cover the following topics:

- Definition and equivalence of integral quadratic forms. Generators of $SL_n(\mathbb{Z})$. Topograph attached to binary quadratic form. Its properties: wells (for definite forms) and rivers (for indefinite forms). Periodicity of rivers.
- Quadratic fields and orders. Class group. Relation between full modules and binary quadratic forms. Bijection of the narrow class group (for a quadratic order of discriminant D) and equivalence classes of binary quadratic forms with the discriminant D . Reduction theory for binary quadratic forms. Purely periodic continued fractions and cycles.
- Multiplication of binary quadratic forms (Gauss law). Bhargava's interpretation in terms of cubes. Cubic binary forms and elements of order 3 in the class group.

Research Objective: The theory of indefinite quadratic forms is more complicated than the theory of definite quadratic forms. For example, while the class number for complex orders (i.e. $D < 0$) is equal to the number of reduced quadratic forms, in the real case (i.e. $D > 0$) the continued fraction algorithm groups reduced forms into cycles, and the the class number is equal to the number of such cycles.

Thus, to every indefinite binary quadratic form we can attach a river period (appearing in the topograph), and a continued fraction period. It would be interesting to understand relation between the two. Second, it is easy to see that elements of order 2 in the class group correspond to cycles with an axial symmetry. How can we characterize elements of order 2 in terms of its river period?

Resources: I will use Conway's book to introduce the topograph. Additional resource is Marty Weissman's "semantic wiki":

http://slugmath.ucsc.edu/mediawiki/index.php/Clust/Binary_quadratic_forms

When there, click on Definitions or Statements. A limited exposition of other topics is contained in notes on my web page (under 2009 REU):

<http://www.math.utah.edu/savin/>

Schedule: I will lecture every day for two hours: 9:30-10:30 and 11:00-12:00. Lunch break is 12:00-2:00. Afternoon session, 2:00-4:00, is dedicated to examples, review, problems (sometimes in the computer lab) as well as helping you with background material, if needed. (Let me know!)

First week classrooms. Morning: MWF LCB 215; Tu JWB 333; Th LCB 222. Afternoon: LCB 115. This will change for second and third weeks.