

MATH 3160-001, SAMPLE FINAL EXAM.

1) Find two complex numbers $z = x + yi$ such that $z^2 = i$.

2) (This is in the book) Let $\omega = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ be a third root of one. Use the theorem on residues to compute

$$\int_0^\infty \frac{1}{z^3 + 1} dz$$

by considering a contour consisting of a segment from $R\omega$ to 0, a segment from 0 to R and then a part of the circle of radius R back to $R\omega$.

3) Find the Laurent series expansion of

$$f(z) = \frac{1}{z^2 - z^3}$$

around 0 and 1.

4) Compute the residue of $\frac{1}{\sin z}$ for $z = k \cdot \pi$ where k is an integer. Then use the theorem on residues to compute

$$\frac{1}{2\pi i} \int_C \frac{1}{\sin z} dz$$

where the integral is computed in the counter clockwise direction along the circle $|z| = 4$.

5) Use a single residue to compute

$$\frac{1}{2\pi i} \int_C \frac{z^6}{z^4 - 1} dz.$$

where the integral is computed in the counter clockwise direction along the circle $|z| = 2$.

6) Show that $z = 0$ is the only zero of the polynomial $z^5 + z^4 + 3z$ inside the circle $|z| = 1$. Use this to compute

$$\frac{1}{2\pi i} \int_C \frac{1}{z^5 + z^4 + 3z} dz.$$

7) Compute

$$\frac{1}{2\pi i} \int_C \frac{5z^4 + 1}{z^5 + z + 1} dz$$

where the integral is computed in the counter clockwise direction along the circle $|z| = 2$. Hint: where are the zeroes of $z^5 + z + 1$ located?

8) How many zeroes does $z^5 - 9z^3 + z^2 + z + 1$ have on $2 < |z| < 4$?