1) Find two complex numbers \( z = x + yi \) such that \( z^2 = i \).

2) (This is in the book) Let \( \omega = -\frac{1}{2} + \frac{\sqrt{3}}{2} i \) be a third root of one. Use the theorem on residues to compute
\[
\int_0^\infty \frac{1}{z^3 + 1} \, dz
\]
by considering a contour consisting of a segment from \( R\omega \) to 0, a segment from 0 to \( R \) and then a part of the circle of radius \( R \) back to \( R\omega \).

3) Find the Laurent series expansion of
\[
f(z) = \frac{1}{z^2 - z^3}
\]
around 0 and 1.

4) Compute the residue of \( \frac{1}{\sin z} \) for \( z = k \cdot \pi \) where \( k \) is an integer. Then use the theorem on residues to compute
\[
\frac{1}{2\pi i} \int_C \frac{1}{\sin z} \, dz
\]
where the integral is computed in the counter clockwise direction along the circle \( |z| = 4 \).

5) Use a single residue to compute
\[
\frac{1}{2\pi i} \int_C \frac{z^6}{z^4 - 1} \, dz.
\]
where the integral is computed in the counter clockwise direction along the circle \( |z| = 2 \).

6) Show that \( z = 0 \) is the only zero of the polynomial \( z^5 + z^4 + 3z \) inside the circle \( |z| = 1 \). Use this to compute
\[
\frac{1}{2\pi i} \int_C \frac{1}{z^5 + z^4 + 3z} \, dz.
\]

7) Compute
\[
\frac{1}{2\pi i} \int_C \frac{5z^4 + 1}{z^5 + z + 1} \, dz
\]
where the integral is computed in the counter clockwise direction along the circle \( |z| = 2 \). Hint: where are the zeroes of \( z^5 + z + 1 \) located?

8) How many zeroes does \( z^5 - 9z^3 + z^2 + z + 1 \) have on \( 2 < |z| < 4 \)?